EECS 455 Problem Set 5

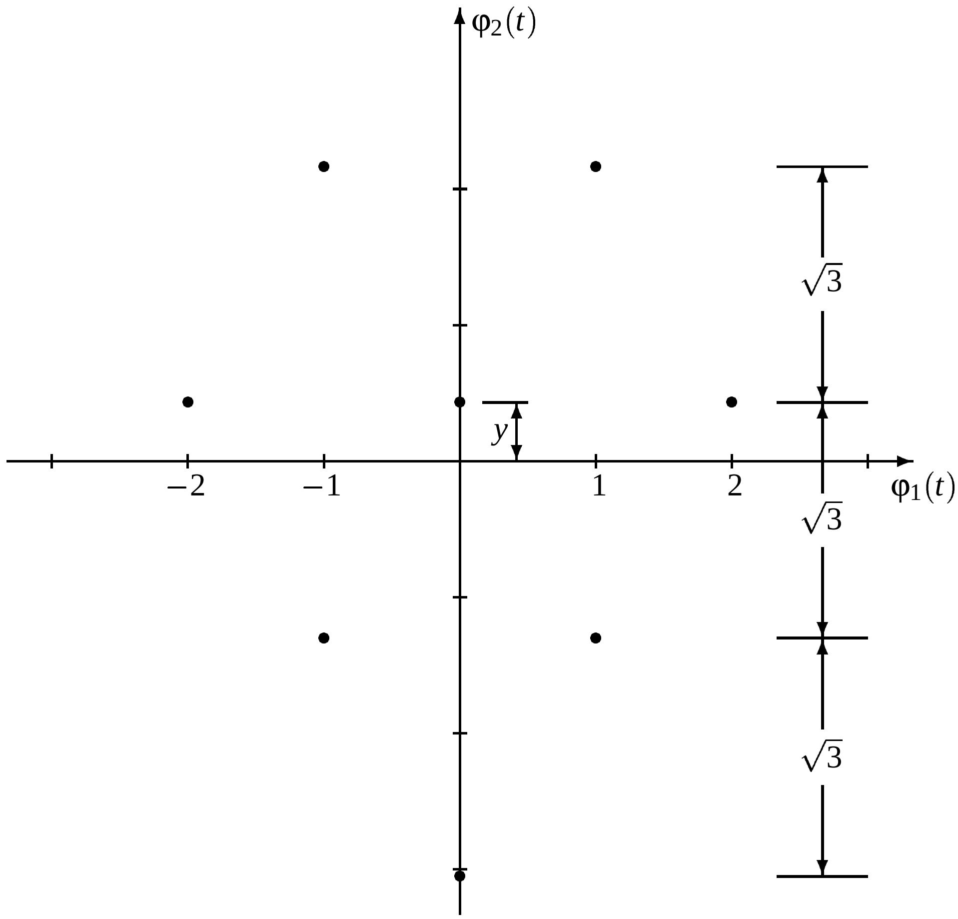
Due: Friday October 15.

1. Consider a binary communications system with $s_i(t) = (-1)^i A p_T(t),\ \ i = 0,1$ and an additive white Gaussian noise (AWGN) channel. Let $h(t)$ be the impulse response of the receiver filter with
$$h(t) = \frac{1}{\sqrt{2\pi}\alpha} \exp\{-t^2/2\alpha^2\}.$$
The noise process $n(t)$ has spectral density $N_0/2$.
   (a) Find the optimal sampling time T_0.
   (b) Find the optimal parameter α. Hint: The maximum with respect to x of $(2\Phi(x)-1)/\sqrt{x}$ occurs at $x = 1.40$ and takes the value .70865.
   (c) Find $P_{e,i}$ for E/N_0= 8,9,10,11,12 dB.
   (d) Compare your answer of part (c) to the error probability for the optimal receiver (filter, sampler, threshold). You may assume that $\pi_0 = \pi_1$.

2. A modulator uses two orthonormal signals ($\phi_1(t)$ and $\phi_2(t)$) to transmit 3 bits of information (8 possible equally likely signals) over an additive white Gaussian noise channel with power spectral density $N_0/2$. The signals are given as
$$\begin{aligned}
s_1(t) &= A(-1\phi_1(t) + (y+\sqrt{3})\phi_2(t)) \\
s_2(t) &= A(1\phi_1(t) + (y+\sqrt{3})\phi_2(t)) \\
s_3(t) &= A(-2\phi_1(t) + (y)\phi_2(t)) \\
s_4(t) &= A(0\phi_1(t) + y\phi_2(t)) \\
s_5(t) &= A(2\phi_1(t) + (y)\phi_2(t)) \\
s_6(t) &= A(-1\phi_1(t) + (y-\sqrt{3})\phi_2(t)) \\
s_7(t) &= A(1\phi_1(t) + (y-\sqrt{3})\phi_2(t)) \\
s_8(t) &= A(0\phi_1(t) + (y-2\sqrt{3})\phi_2(t))
\end{aligned}$$

   Determine the optimum value of the parameter y to minimize the average signal energy transmitted.

   Draw the optimum decision regions for the signal set (in two dimensions).

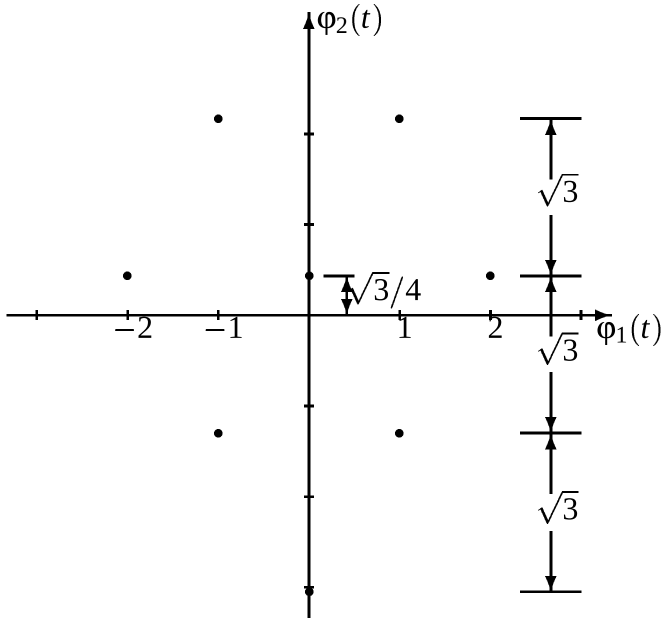


3. A modulator uses two orthonormal signals ($\phi_1(t)$ and $\phi_2(t)$) to transmit 3 bits of information (8 possible equally likely signals) over an additive white Gaussian noise channel with power spectral density $N_0/2$. The signals are given as

$$\begin{aligned}
s_1(t) &= A(-1\phi_1(t) + (y+\sqrt{3})\phi_2(t)) \\
s_2(t) &= A(1\phi_1(t) + (y+\sqrt{3})\phi_2(t)) \\
s_3(t) &= A(-2\phi_1(t) + (y)\phi_2(t)) \\
s_4(t) &= A(0\phi_1(t) + y\phi_2(t)) \\
s_5(t) &= A(2\phi_1(t) + (y)\phi_2(t)) \\
s_6(t) &= A(-1\phi_1(t) + (y-\sqrt{3})\phi_2(t)) \\
s_7(t) &= A(1\phi_1(t) + (y-\sqrt{3})\phi_2(t)) \\
s_8(t) &= A(0\phi_1(t) + (y-2\sqrt{3})\phi_2(t))
\end{aligned}$$

Simulate the error probability of this system in additive white Gaussian noise. Use the optimum value of the parameter y that minimizes the average signal energy transmitted (from previous homework).

Vary E_b/N_0 from 0dB to 10dB where $E_b = \bar{E}/3$ is the energy per bit of information. Plot the average symbol error probability versus E_b/N_0 in dB on a log-linear scale.



$\varphi_2(t)$
$\sqrt{3}$
$\sqrt{3}/4$
-2
-1
1
2
$\varphi_1(t)$
$\sqrt{3}$
$\sqrt{3}$