## EECS 455 Problem Set 6

Due: Wednesday November 2.

1. Consider a communication system that transmits one of 16 signals in 7 dimensions. The signals are

$$\begin{array}{rcl} s_{0} &=& \sqrt{E}(+1,+1,+1,+1,+1,+1,+1) \\ s_{1} &=& \sqrt{E}(+1,+1,-1,-1,-1,-1) \\ s_{2} &=& \sqrt{E}(+1,+1,-1,-1,-1,-1,+1) \\ s_{3} &=& \sqrt{E}(+1,+1,-1,-1,-1,-1,+1,-1) \\ s_{4} &=& \sqrt{E}(+1,-1,+1,+1,-1,-1,-1,-1) \\ s_{5} &=& \sqrt{E}(+1,-1,-1,+1,+1,-1,-1) \\ s_{6} &=& \sqrt{E}(+1,-1,-1,-1,+1,+1,+1,-1) \\ s_{7} &=& \sqrt{E}(+1,-1,-1,-1,+1,+1,-1,-1) \\ s_{8} &=& \sqrt{E}(-1,+1,+1,+1,-1,-1,-1,+1) \\ s_{9} &=& \sqrt{E}(-1,+1,-1,+1,+1,-1,-1) \\ s_{10} &=& \sqrt{E}(-1,+1,-1,-1,+1,+1,-1) \\ s_{11} &=& \sqrt{E}(-1,-1,+1,+1,+1,-1,-1) \\ s_{12} &=& \sqrt{E}(-1,-1,-1,-1,+1,+1,+1) \\ s_{13} &=& \sqrt{E}(-1,-1,-1,-1,-1,-1,-1) \\ s_{14} &=& \sqrt{E}(-1,-1,-1,-1,-1,-1,-1) \\ s_{15} &=& \sqrt{E}(-1,-1,-1,-1,-1,-1,-1) \end{array}$$

Four bits are transmitted each with energy 7E so the energy per bit is

$$E_b = 7E/4.$$

We will consider two receivers.

(a) The first receiver is the optimal receiver for minimizing the probability of signal error (choosing the wrong signal). For this receiver we want to determine the probability of bit error. Assume the 4 bits that are transmitted are represented in binary and the binary number determines the signal. That is the bits 0000 represent 0 and are mapped to  $s_0$ . The bits 0001 represent 1 and are mapped to  $s_1$ . The bits 0101 represent 5 and are mapped to  $s_5$ . Simulate this communication system in additive white Gaussian noise (power spectral density  $N_0/2$  to determine the bit error probability. Plot  $P_e$  versu  $E_b/N_0(dB)$  as  $E_b/N_0(dB)$  varies from 0 dB to 7dB.

(b) The second receiver computes the decision variable corresponding to each dimension  $\tilde{r}_i = \operatorname{sign}(r_i)$  and then finds the signal vector that has the fewest differences (number of positions where an error occurs). Simulate this system as well. Determine the bit error probability as a function of the energy per information bit. Plot  $P_e$  versu  $E_b/N_0(dB)$  as  $E_b/N_0(dB)$  varies from 0 dB to 8dB.