Lecture 11

Goals:

- Be able to modulate and demodulate an OQPSK signal
- Be able to modulate and demodulate an MSK signal
- Be able to compare performance (bandwidth efficiency and energy efficiency)

In this lecture we examine a number of different simple modulation schemes. We examine the implementation of the optimum receiver, the error probability and the bandwidth occupancy. We would like the simplest possible receiver, with the lowest error probability and smallest bandwidth for a given data rate.

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In this lecture we illustrate one main drawback to BPSK. The fact that the signal amplitude has discontinuities causes the spectrum to have fairly large sidelobes. For a system that has a constraint on the bandwidth this can be a

problem. A possible solution is to filter the signal. A bandpas filter centered at the carrier frequency which removes the sidbands can be inserted after mixing to the carrier frequency. Alternatly we can filter the data signal at baseband before mixing to the carrier frequency.

Below we simulate this type of system to illustrate the effect of filtering and nonlinear amplification. The data waveform b(t) is mixed onto a carrier. This modulated waveform is denoted by

$$s_1(t) = b(t)\cos(2\pi f_c t)$$

$$s_2(t) = \int h(t-\tau)s_1(\tau)d\tau$$

$$s_2(t) = \int h(t-\tau)b(\tau)\cos(2\pi f_c\tau)d\tau$$

Consider the case where the signal $s_1(t)$ is filtered by a fourth order bandpass Butterworth filter with passband from $f_c - 4R_b$ to $f_c + 4R_b$ The filtered signal

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is denoted by $s_2(t)$. The signal $s_2(t)$ is then amplified. The input-output characteristics of the amplifier are

$$s_3(t) = 100 \tanh(2s_1(t))$$

This amplifier is fairly close to a hard limiter in which every input greater than zero is mapped to 100 and every input less than zero is mapped to -100.

Simulation Parameters

Sampling Frequency= 50MHz Sampling Time =20nseconds Center Frequency= 12.5MHz Data Rate=390.125kbps Simulation Time= 1.31072 m s













Quaternary Phase Shift Keying (QPSK)

The next modulation technique we consider is QPSK. In this modulation technique one of four phases of the carrier is transmitted in a symbol duration denoted by T_s . Since one of four waveforms is transmitted there are two bits of information transmitted during each symbol duration. An alternative way of describing QPSK is that of two carriers offset in phase by 90 degrees. Each of these carriers is then modulated using BPSK. These two carriers are called the inphase and quadrature carriers. Because the carriers are 90 degrees offset, at the output of the correlation receiver they do not interfer with each other (assuming perfect phase synchronization). The advantage of QPSK over BPSK is that the the data rate is twice as high for the same bandwidth. Alternatively single-sideband BPSK would have the same rate in bits per second per hertz but would have a more difficult job of recovering the carrier frequency and phase.

 $\sqrt{P}\cos(2\pi f_c t)$ $b_c(t)$ $b_s(t)$ $b_s(t)$ $-\sqrt{P}\sin(2\pi f_c t)$

Figure 54: Modulator for QPSK

$$b_c(t) = \sum_{l=-\infty}^{\infty} b_{c,l} p_{T_s}(t-lT_s), \ b_{c,l} \in \{+1,-1\}$$

$$p_s(t) = \sum_{l=-\infty}^{\infty} b_{s,l} p_{T_s}(t-lT_s), \quad b_{s,l} \in \{+1,-1\}$$

$$s(t) = \sqrt{P}[b_c(t)\cos(2\pi f_c t) - b_s(t)\sin(2\pi f_c t)]$$

= $\sqrt{2P}\cos(2\pi f_c t + \phi(t))$

The transmitted power is still *P*. The symbol duration is T_s seconds. The data rate is $R_b = 2/T_s$ bits seconds.

The phase $\phi(t)$, of the transmitted signal is related to the data waveform as follows.

$$\phi(t) = \sum_{l=-\infty}^{\infty} \phi_l \ p_{T_s}(t - lT_s), \quad \phi_l \in \{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$$

The relation between ϕ_l and $b_{c,l}, b_{s,l}$ is shown in the following table

$b_{c,l}$	$b_{s,l}$	φ _l
+1	+1	$\pi/4$
-1	+1	3π/4
-1	-1	5π/4
+1	-1	$7\pi/4$



The constellation of QPSK is shown below. The phase of the overall carrier can be on of four values. Transitions between any of the four values may occur at any symbol transition. Because of this, it is possible that the transition is to the 180 degree opposite phase. When this happens the amplitude of the signal goes through zero. In theory this is an instantaneous transition. In practice, when the signal has been filtered to remove out-of-band components this transition is slowed down. During this transition the amplitude of the carrier goes through zero. This can be undesireable for various reasons. One reason is that nonlinear amplifiers with a non constant envelope signal will regenerate the out-of-band spectral components. Another reason is that at the receiver, certain synchronization circuits need constant envelope to maintain their tracking capability.



The bandwidth of QPSK is given by

$$S(f) = PT_s/2 [\operatorname{sinc}^2((f - f_c)T_s) + \operatorname{sinc}^2((f + f_c)T_s)]$$

$$= PT_b \left[\operatorname{sinc}^2 (2(f - f_c)T_b) + \operatorname{sinc}^2 (2(f + f_c)T_b) \right]$$

since $T_s = T_b/2$. Thus while the spectrum is compressed by a factor of 2 relative to BPSK with the same bit rate, the center lobe is also 3dB higher, that is the peak power density is higher for QPSK than BPSK. The null-to-null bandwidth is $2/T_s = R_b$.











Assuming $2\pi f_c T_s = 2\pi n$ or $2\pi f_c T_s \gg 1$

$$X_c(iT_s) = \sqrt{PT_s/2} \ b_{c,i-1} + \eta_{c,i} = \sqrt{E_b} \ b_{c,i-1} + \eta_{c,i}$$

$$X_s(iT_s) = \sqrt{PT_s/2} b_{s,i-1} + \eta_{s,i} = \sqrt{E_b} b_{s,i-1} + \eta_{s,i}$$

where $E_b = PT_s/2$ is the energy per transmitted bit. Also $\eta_{c,i}$ and $\eta_{s,i}$ are Gaussian random variables, with mean 0 and variance $N_0/2$.

Bit Error Probability of QPSK

$$P_{e,b} = Q(\sqrt{\frac{2E_b}{N_0}})$$

The probability that a symbol error is made is

$$P_{e,s} = 1 - (1 - P_{e,b})^2 = 2P_{e,b} - P_{e,b}^2$$

Thus for the same data rate, transmitted power, and bit error rate (probability of error), QPSK has half the (null-to-null) bandwidth of BPSK.

Example

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Given:

- Noise power spectral density of $N_0/2 = -110 \text{ dBm/Hz} = 10^{-14} \text{ Watts/Hz}.$
- $P_r = 3 \times 10^{-6}$ Watts
- Desired $P_e = 10^{-7}$.

Find: The data rate that can be used and the bandwidth that is needed for QPSK.

Solution: Need $Q(\sqrt{2E_b/N_0}) = 10^{-7}$ or $E_b/N_0 = 11.3$ dB or $E_b/N_0 = 13.52$. But

$$E_b/N_0 = \frac{\frac{P_r}{2}(T_s)}{N_0} = P_r T/N_0 = 13.52$$

since $T_s = 2T$. Thus the data bit must be at least $T = 9.0 \times 10^{-8}$ seconds long, i.e. the data rate 1/T must be less than 11 Mbits/second. Clearly we also need a (null-to-null) bandwidth of 11 MHz.

Offset Quaternary Phase Shift Keying (OQPSK)

The disadvantages of QPSK can be fixed by offsetting one of the data streams by a fraction (usually 1/2) of a symbol duration. By doing this we only allow one data bit to change at a time. When this is done the possible phase transitions are \pm 90 deg. In this way the transitions through the origin are illiminated. Offset QPSK then gives the same performance as QPSK but will have less distorition when there is filtering and nonlinearities.



$$b_{c}(t) = \sum_{l=-\infty}^{\infty} b_{c,l} p_{T_{s}}(t-lT_{s}), \quad b_{c,l} \in \{+1,-1\}$$

$$b_{s}(t) = \sum_{l=-\infty}^{\infty} b_{s,l} p_{T_{s}}(t-lT_{s}), \quad b_{s,l} \in \{+1,-1\}$$

$$s(t) = \sqrt{P}[b_{c}(t-T_{s}/2)\cos(2\pi f_{c}t) - b_{s}(t)\sin(2\pi f_{c}t)]$$

$$s(t) = \sqrt{2P}\cos(2\pi f_{c}t + \phi(t))$$

The transmitted power is still *P*. The symbols duration is T_s seconds. The data rate is $R_b = 2/T_s$ bits seconds. The bandwidth (null-to-null) is $2/T_s = R_b$. This modification of QPSK removes the possibility of both data bits changing simultaneously. However, one of the data bits may change every $T_s/2$ seconds but 180 degree changes are not allowed. The bandwidth of OQPKS is the same as QPSK. OQPSK has advantage over QPSK when passed through

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nonlinearities (such as in a satellite) in that the out of band interference generated by first bandlimiting and then hard limiting is less with OQPSK than QPSK.





Assuming $2\pi f_c T_s = 2\pi n$ or $2\pi f_c T_s \gg 1$

$$\begin{aligned} X_c(iT_s - T_s/2) &= \sqrt{PT_s/2} \ b_{c,i-1} + \eta_{c,i} = \sqrt{E_b} \ b_{c,i-1} + \eta_{c,i} \\ X_s(iT_s) &= \sqrt{PT_s/2} \ b_{s,i-1} + \eta_{s,i} = \sqrt{E_b} \ b_{s,i-1} + \eta_{s,i} \end{aligned}$$

where $E_b = PT_s/2$ is the energy per transmitted bit. Also $\eta_{c,i}$ and $\eta_{s,i}$ are Gaussian random variables, with mean 0 variance $N_0/2$.

Bit Error Probability of OQPSK

$$P_{e,b} = Q(\sqrt{\frac{2E_b}{N_0}})$$

The probability that a symbol error is made is

$$P_{e,s} = 1 - (1 - P_{e,b})^2 = 2P_{e,b} - P_{e,b}^2$$

This is the same as QPSK.

Minimum Shift Keying (MSK)

Minimum shift keying can be viewed in several different ways and has a number of significant advantages over the previously considered modulation schemes. MSK can be thought of as a variant of OQPSK where the data pulse waveforms are shaped to allow smooth transition between phases. It can also be thought of a a form of frequency shift keying where the two frequencies are separated by the minimum amount to maintain orthogonality and have continuous phase when switching from one frequency to another (hence the name minimum shift keying). The advantages of MSK include a better spectral efficiency in most cases. In fact the spectrum of MSK falls off at a faster rate than BPSK, QPSK and OQPSK. In addition there is an easier implementation than OQPSK (called serial MSK) that avoids the problem of having a precisely controlled time offset between the two data streams. An additional advantage is that MSK can be demodulator noncoherently as well as coherently. So for applications requiring a low cost receiver MSK may be a



$$b_{c}(t) = \sum_{l=-\infty}^{\infty} b_{c,l} p_{T_{s}}(t-lT_{s}), \quad b_{c,l} \in \{+1,-1\}$$

$$b_{s}(t) = \sum_{l=-\infty}^{\infty} b_{s,l} p_{T_{s}}(t-lT_{s}), \quad b_{s,l} \in \{+1,-1\}$$

$$c(t) = \sqrt{2} \sin(\pi t/T_{s}) \quad c(t-T_{s}/2) = -\sqrt{2} \cos(\pi t/T_{s})$$

$$s(t) = \sqrt{P} [b_{c}(t-T_{s}/2)c(t-T_{s}/2)\cos(2\pi f_{c}t) - b_{s}(t)c(t)\sin(2\pi f_{c}t)]$$

$$s(t) = \sqrt{2P} [\{-b_{c}(t-T_{s}/2)\cos(\pi t/T_{s})\}\cos(2\pi f_{c}t) - \{b_{s}(t)\sin(\pi t/T_{s})\}\sin(2\pi f_{c}t)]$$

$$= \sqrt{2P} \cos(2\pi f_{c}t + \phi(t))$$
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$b_c(t-T_s/2)$	$b_s(t)$	$\phi(t)$
+1	+1	$\pi - \frac{\pi t}{T_s}$
+1	-1	$\pi + \frac{\pi t}{T_s}$
-1	+1	$\frac{\pi t}{T_s}$
-1	-1	$-\frac{\pi t}{T_s}$

In the above table, because of the delay of the bit stream corresponding to the cosine branch, only one bit is allowed to change at a time. During each time interval of duration $T_s/2$ during which the data bits remain constant there is a phase shift of $\pm \pi/2$. Because the phase changes linearly with time MSK can also be viewed as frequency shift keying. The two different frequencies are $f_c + \frac{1}{2T_s}$ and $f_c - \frac{1}{2T_s}$. The change in frequency is $\Delta f = \frac{1}{T_s} = \frac{1}{2T_b}$ where $T_b^{-1} = 2/T_s$ is the data bit rate. The transmitted power is still *P*. The symbols duration is T_s seconds. The data rate is $R_b = 2/T_s$ bits seconds. The signal has constant envelope which is useful for nonlinear amplifiers. The bandwidth is different because of the pulse shaping waveforms.

where

$$\cos(\phi(t)) = -b_c(t-T_s/2)\cos(\pi t/T_s)$$

$$\sin(\phi(t)) = b_s(t)\sin(\pi t/T_s)$$

$$\phi(t) = \tan^{-1} \left(\frac{b_s(t)\sin(\pi t/T_s)}{-b_c(t-T_s/2)\cos(\pi t/T_s)} \right)$$







The spectrum of MSK is given by

$$S(f) = \frac{8PT_b}{\pi^2} \left\{ \frac{\cos^2(2\pi T_b(f - f_c))}{[1 - (4T_b(f - f_c))^2]^2} + \frac{\cos^2(2\pi T_b(f + f_c))}{[1 - (4T_b(f + f_c))^2]^2} \right\}$$

The nulls in the spectrum are at $(f - f_c)T_b = 0.75, 1.25, 1.75,...$ Because we force the signal to be continuous in phase MSK has significantly faster decay of the power spectrum as the frequency from the carrier becomes larger. MSK decays as $1/f^4$ while QPSK, OQPSK, and BPSK decay as $1/f^2$ as the frequency differs more and more from the center frequency.









Assuming $2\pi f_c T_s = 2\pi n$ or $2\pi f_c T_s \gg 1$

$$X_{c}(iT_{s}+T_{s}/2) = \sqrt{PT_{s}/2} b_{c,i-1} + \eta_{c,i} = \sqrt{E_{b}} b_{c,i-1} + \eta_{c,i}$$
$$X_{s}(iT_{s}) = \sqrt{PT_{s}/2} b_{s,i-1} + \eta_{s,i} = \sqrt{E_{b}} b_{s,i-1} + \eta_{s,i}$$

where $E_b = PT_s/2$ is the energy per transmitted bit. Also $\eta_{c,i}$ and $\eta_{s,i}$ are Gaussian random variables, with mean 0 variance $N_0/2$.

Bit Error Probability of MSK with Coherent Demodulation Since the signals are still antipodal

$$P_{e,b} = Q(\sqrt{\frac{2E_b}{N_0}})$$

The probability that a symbol error is made is

$$P_{e,s} = 1 - (1 - P_{e,b})^2 = 2P_{e,b} - P_{e,b}^2$$









Noncoherent Demodulation

Because MSK can be viewed as a form of Frequency Shift Keying it can also be demodulated noncoherently. For the same sequence of data bits the frequency is $f_c - 1/2T_s$ if $b_c(t - T_s/2) = b_s(t)$ and is $f_c + 1/2T_s$ if $b_c(t - T_s/2) \neq b_s(t)$.

Consider determining $b_{s,i-1}$ at time $(i - 1/2)T_s$. Assume we have already determined $b_{c,i-2}$ at time $(i - 1)T_s$. If we estimate which of two frequencies is sent over the interval $[(i - 1)T_s, (i - 1/2)T_s)$ the decision rule is to decide that $b_{s,i-1} = b_{c,i-2}$ if the frequency detected is $f_c - 1/(2T_s)$ and to decide that $b_{s,i-1} = -b_{c,i-2}$ if the frequency detected is $f_c + 1/(2T_s)$.

Consider determining $b_{c,i-1}$ at time iT_s . Assume we have already determined $b_{s,i-1}$ at time $(i-1/2)T_s$. If we estimate which of two frequencies is sent over the interval $[(i-1/2)T_s, iT_s)$ the decision rule is to decide that $b_{c,i-1} = b_{s,i-1}$ if the frequency detected is $f_c - 1/(2T_s)$ and to decide that $b_{c,i-1} = -b_{s,i-1}$ if the frequency detected is $f_c + 1/(2T_s)$.



The method to detect which of the two frequencies is transmitted is identical to that of Frequency Shift Keying which will be considered later.

For the example phase waveform shown previously we have that

Time Interval	$[0, T_s/2)$	$[T_s/2,T_s)$	$[T_s, 3T_s/2)$	$[3T_s/2, 2T_s)$	$[2T_s, 5T_s/2)$
Frequency	+	+	—	+	—
Previous Data	$b_{c,-1} = +1$	$b_{s,0} = -1$	$b_{c,0} = 1$	$b_{s,1} = +1$	$b_{c,1} = -1$
Detected Data	$b_{s,0} = -1$	$b_{c,0} = +1$	$b_{s,1} = +1$	$b_{c,1} = -1$	$b_{s,2} = -1$

So detecting the frequency can also be used to detect the data.

Serial Modulation and Demodulation

The implementation of MSK as parallel branches suffers from significant sensitivity to precise timing of the data (exact shift by T for the inphase component) and the exact balance between the inphase and quadriphase carrier signals. An alternative implementation of MSK that is less complex and does not have these draw backs is known as serial MSK. Serial MSK does

have an additional restriction that $f_c = (2n+1)/4T$ which may be important when f_c is about the same as 1/T but for $f_c \gg 1/T$ it is not important. The block diagram for serial MSK modulator and demodulator is shown below.



where $f_1 = f_c - \frac{1}{4T}$ and $f_2 = f_c + \frac{1}{4T}$. (For serial MSK we require $f_c = (2n+1)/4T$ for some integer *n*. Otherwise the implementation does not give constant envelope).

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The filter H(f) is given by

$$H(f) = \frac{4T}{\pi} \frac{\cos[2\pi(f-f_1)T - 0.25]}{1 - 16[(f-f_1)T - 0.25]^2} e^{-j2\pi(f-f_1)T}$$

The low pass filter (LPF) removes double frequency components. Serial MSK is can also be viewed as a filtered form of BPSK where the BPSK signal center frequency is f_1 but the filter is not symmetric with respect to f_1 . The receiver is a filter matched to the transmitted signal (and hence optimal). The output is then mixed down to baseband where it is filtered (to remove the double frequency terms) and sampled.

MSK is a special case of a more general form of modulation known as continuous phase modulation where the phase is continuous. The general form of CPM is given by

$$s(t) = \sqrt{2P}\cos(2\pi f_c t + \phi(t))$$

where the phase waveform has the form

$$\begin{aligned} \phi(t) &= 2\pi h \int_0^t \sum_{i=0}^k b_i g(\tau - iT) d\tau + \phi_0 \quad kT \le t \le (k+1)T \\ &= 2\pi h \sum_{i=0}^k b_i q(t - iT) + \phi_0 \quad kT \le t \le (k+1)T \end{aligned}$$

The function $g(\cdot)$ is the (instantaneous) frequency function, *h* is called the modulation index and b_i is the data. The function $q(t) = \int_0^t g(\tau) d\tau$ is the phase waveform. The function $g(t) = \frac{dg(t)}{dt}$ is the frequence waveform.

For example if CPM has h = 1/2 and

$$q(t) = \begin{cases} 0, & t < 0\\ t/2, & 0 \le t < T\\ 1/2, & t > T. \end{cases}$$

then the modulation is the same as MSK. Continuous Phase Modulation Techniques have constant envelope which make them useful for systems involving nonlinear amplifiers which also must have very narrow spectral widths.

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Example

- Noise power spectral density of $N_0/2 = -110 \text{ dBm/Hz} = 10^{-14} \text{ Watts/Hz}.$
- $P_r = 3 \times 10^{-6}$ Watts

Given:

- Desired $P_e = 10^{-7}$.
- Bandwidth available=26MHz (at the 902-928MHz band). The peak power outside must be 20dB below the peak power inside the band.

Find: The data rate that can be used for MSK.

Solution: Need $Q(\sqrt{2E_b/N_0}) = 10^{-7}$ or $E_b/N_0 = 11.3$ dB or $E_b/N_0 = 13.52$. But $E_b/N_0 = P_r T/N_0 = 13.52$. Thus the data bit must be at least $T = 9.0 \times 10^{-8}$ seconds long, i.e. the data rate 1/T must be less than 11 Mbits/second.

Gaussian Minimum Shift Keying

Gaussian minimum shift keying is a special case of continuous phase modulation discussed in the previous section. For GMSK the pulse waveforms are given by

$$g(t) = Q(\frac{t-T}{\sigma}) - Q(\frac{t}{\sigma})$$



Figure 77: Phase Waveform for Gaussian Minimum Shift Keying_{X1-65} (BT=0.3)













Figure 82: Waveform for Gaussian Minimum Shift Keying_{XI-70} (BT=0.3)





Figure 85: Waveform for Gaussian Minimum Shift Keying_{XI-73} (BT=0.3)



$\pi/4$ QPSK

As mentioned earlier the effect of filtering and nonlinearly amplifying a QPSK waveform causes distortion when the signal amplitude fluctuates significantly. Another modulation scheme that has less fluctuation that QPSK is $\pi/4$ QPSK. In this modulation scheme every other symbol is sent using a rotated (by 45 degrees) constellation. Thus the transitions from one phase to the next are still instantaneous (without any filtering) but the signal never makes a transition through the origin. Only ± 45 and ± 135 degree transitions are possible. This is shown in the constellation below where a little bit of filtering was done.





