EECS 461, Fall 2009, Problem Set 3[1]

issued: Thursday October 1, 2009 due: Thursday October 8, 2009

In this problem set you will use Matlab to obtain several plots. My major concern is that you understand these plots, so please include explanation of each plot in your discussion!

1. Consider a low pass filter with frequency response

$$H(j\omega) = \frac{1}{j\omega\tau + 1}.$$

   (a) Using the Matlab file prob1_ps3.m, plot the frequency response of this filter for $\tau = 0.5$.
   (b) Using the frequency response plot, determine $y_{ss}(t)$, the sinusoidal steady state response of the filter output, to an input signal $u(t) = \sin(20t)$.
   (c) Consider two sinusoids with the same frequency ω_0: $u_1(t) = A_1 \sin(\omega_0 t)$ and $u_2(t) = A_2 \sin(\omega_0 t + \phi)$. At what times do each of these sinusoids cross zero? Let t_1 and t_2 denote the times of corresponding zero crossings for the two sinusoids. Compute $\Delta t = t_2 - t_1$ in terms of ω_0 and ϕ.
   (d) Using Matlab, plot the time response of the filtered output in response to the input $u(t) = \sin(20t)$. Does the time response agree with the result predicted using the frequency response plot? Check both the magnitude and phase difference; as shown in part (1c) the latter may be computed from the time between zero crossings.
   (e) How do the answers above change for different values of τ, for example, $\tau = 0.05$ and $\tau = 5$? NOTE: You may need to change the time interval for the simulation so that the transient response has time to die out.

   HAND IN: At least one set of frequency and time response plots, and be very careful to explain what happens with the other plots. If you just hand in plots, do explain why they look the way they do and why you see the trends you see. This problem is fundamental to understanding linear filtering and the relation between bandwidth and frequency response.

   NOTE: The Matlab command "ginput" is useful in picking numerical values from Matlab figures. (Type "[x,y]=ginput" and hit return – you can then pick points off the current figure with the crosshairs until you hit return again.)

2. Consider a DC motor, as shown in Figure 1. The DC motor in the (2008) EECS 461 lab has parameter values: $K_M = 0.0389$ N-m/A, $K_V = 0.0388$ V/(rad/sec), $R = 1.23$ ohms, $L = 0.00034$ H, $J = 6.55 \times 10^{-6}$ N-m/(rad/sec)^2, and $B = 0.00131$ N-m/(rad/sec).

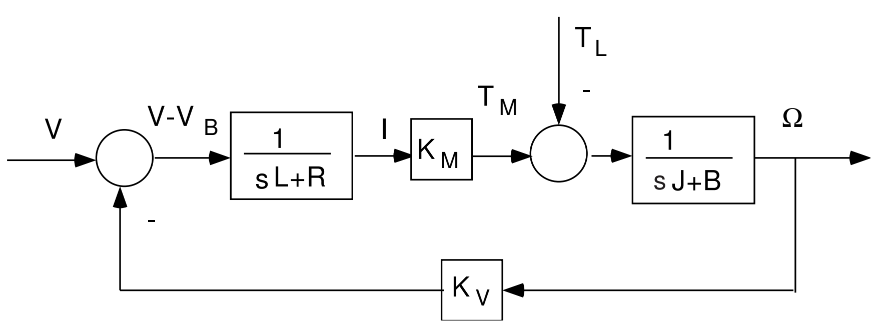

Figure 1: DC Motor

[1] Revised September 30, 2009.

(a) Suppose that this motor is driven by a constant voltage source V and undergoes a constant load T_L. Then, as we showed in Lecture 5, the steady state torque and speed are related by

$$T_M + \left(\frac{K_M K_V}{R}\right)\Omega = \left(\frac{K_M}{R}\right)V$$

Using the Matlab file prob2a_ps3.m, sketch the associated torque speed curves for several values of the input voltage V. Why do these curves not depend on the inductance L or the inertia J?

(b) For a given input voltage V, the specific values of steady state torque and speed will depend on the value of the friction coefficient B and the constant load torque T_L, according to the relations

$$\Omega = \frac{K_M V - RT_L}{K_M K_V + RB}, \qquad T_M = \frac{K_M(BV + K_V T_L)}{K_M K_V + RB}.$$

Using the Matlab file prob2b_ps3.m, compute the steady state values of speed and torque for several values of constant load torque, and plot them on the torque-speed curves. Hand in the plot of torque-speed curves with the different load torques identified.

(c) Use the Matlab file prob2c_ps3.m to plot the frequency responses of speed (in Hz) and torque (in N-m) to a voltage input (in V). What are the DC gains of each response? Do these agree with the torque-speed equation in part (2a)? Why or why not? Explain. Hand in the frequency response plot.

(d) Develop a SIMULINK model of this motor, as shown in Figure 2. Use a "step" block to generate the constant voltage input. Use your model, in conjunction with the Matlab file prob2d_ps3.m to verify

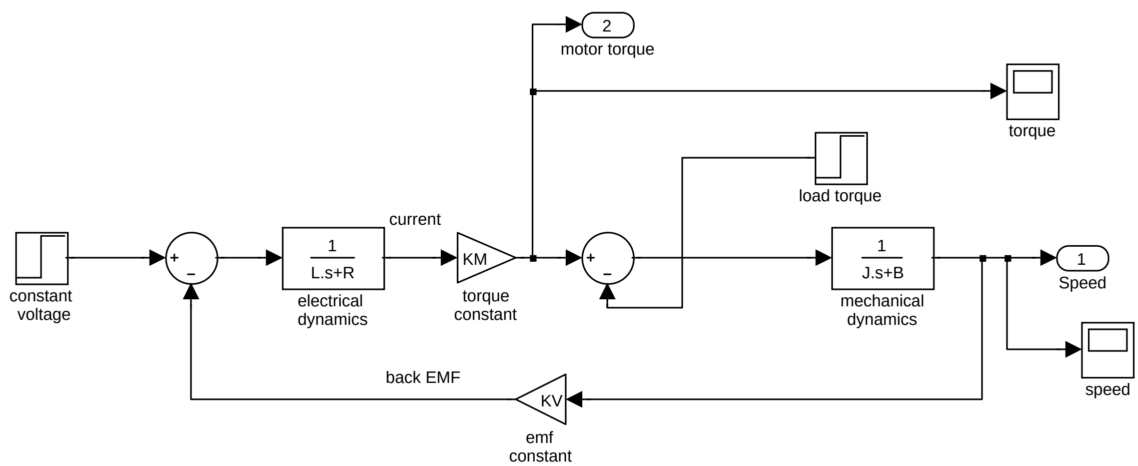

Figure 2: DC Motor with Load Torque Disturbance

that the torque speed curve you constructed agrees with your model. (To do so, run your simulation with several different values of the input voltage and the constant load torque.) Hand in a time response plot for $V = 10$ and $T_L = 0.2$.

(e) Now modify your Simulink model to use a PWM signal to drive the motor; to do so, use a "pulse generator" block to create a PWM signal with a constant duty cycle that alternates between 0 and 5 V. (You need not supply a torque disturbance in this part of the problem, so you can eliminate this block, or set the torque disturbance to zero.) The frequency spectrum of a PWM signal alternating between 0 and 5 will have a nonzero DC component. The gain that this frequency component will see is obtained from the frequency response at $\omega = 0$. The gain seen by the first harmonic is obtained from the frequency response plots evaluated at the switching frequency $f = 1/T$ Hz. By inspecting your plots from part (2c), you can determine the relative contribution to the steady state speed and torque signals from the two frequency components of the PWM signal. Use your Simulink model together with the Matlab file prob2e_ps3.m to plot the responses of the motor speed and torque to a 25kHz PWM signal with a 50% duty cycle. Hand in this plot. Are these plots consistent with the frequency response plot from part (2c)? Explain. Why does the torque exhibit a greater response to the PWM switching frequency than does the speed? Explain.

(f) Examine the speed and torque responses to the PWM input, and determine the steady state value in each case (strictly speaking neither response reaches a steady state due to the high frequency switching noise, so use approximate values.) Show how these values may be computed by using the amplitude and duty cycle of the PWM signal and the DC gains of each response. Verify that the steady state values of speed and torque satisfy the torque speed equation in part (2a) for an input $V = 2.5$ volts.

3. Consider again the DC motor in Figure 1. We showed in class that the transfer functions from V to Ω and T_M are given by

$$\Omega(s) = \frac{\frac{K_M}{(sL+R)(sJ+B)}}{1 + \frac{K_M K_V}{(sL+R)(sJ+B)}} V(s) \tag{1}$$

$$T_M(s) = \frac{\frac{K_M}{(sL+R)}}{1 + \frac{K_M K_V}{(sL+R)(sJ+B)}} V(s). \tag{2}$$

The DC gains of these transfer functions were used in Problem 2 to verify those points on the torque-speed curves for the motor that corresponded to zero load torque. In the present problem, we will see how to verify the torque/speed curves for nonzero load torque.

(a). Use the rules for combining transfer functions from Lecture 6 to compute the transfer functions from T_L to Ω and T_M.

(b). Modify the m-file from Problem 2c to compute the frequency responses and DC gains of these transfer functions.

(c). Denote the transfer functions from V to Ω and T_M by $H_{\Omega V}$ and $H_{T_M V}$, and those from T_L to Ω and T_M by $H_{\Omega T_L}$ and $H_{T_M T_L}$. Linearity implies that the Laplace transforms of Ω and T_M satisfy

$$\Omega(s) = H_{\Omega V}(s)V(s) + H_{\Omega T_L}(s)T_L(s) \tag{3}$$

$$T_M(s) = H_{T_M V}(s)V(s) + H_{T_M T_L}(s)T_L(s). \tag{4}$$

It follows that the steady state response of (say) Ω to a constant voltage input *and* a constant load torque is the sum of the individual responses. Hence knowing the DC gains of the four transfer functions $H_{\Omega V}$, $H_{\Omega T_L}$, $H_{T_M V}$, and $H_{T_M T_L}$ allows us to compute the steady state responses of Ω and T_M to V and T_L, and thus to verify the torque-speed curves for nonzero load torque. Do so for $V = 10$ and $T_L = 0.2$. Are the answers you obtain consistent with the torque-speed curves and the time response plots from Problem 2? Why or why not?

4. In part because A/D converters are constructed from electronic components that may not satisfy specifications precisely, and whose behavior may depend on factors such as ambient temperature, the mapping from analog input voltage to binary output code may not be accurate. Consider the "ideal transfer function" of an ADC depicted in Figure 3. This figure depicts the relation between a voltage input to the ADC and the binary code output from the ADC and expressed an equivalent decimal number. In theory, with infinite precision this function will be a straight line with slope one that intersects the origin. Of course the best we can do with finite wordlength is to make a staircase approximation to this straight line. In practice, the reasons described above imply that this staircase approximation may end up being based around a straight line whose slope differs from one, and that has a nonzero offset from the origin. See Freescale Application Note AN2989 for further discussion.

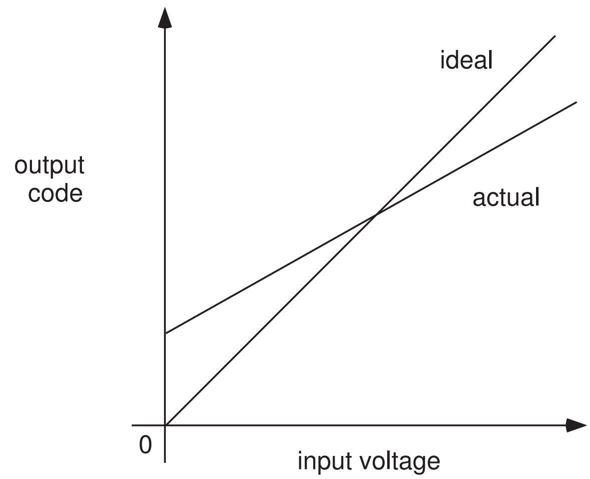

Figure 3: Ideal and Actual Transfer Functions of A/D Converter

Suppose that it is possible to determine values of the actual slope and offset of an ADC. Then, as discussed in AN2989, it is possible to correct for differences between these values and the ideal values of unity slope and zero offset. To do this, it is necessary to have available two known reference voltages, say $V_1 = 25\% V_{max}$ and $V_2 = 75\% V_{max}$, where $(0, V_{max})$ is the range of input voltages to be converted. By comparing the converted values of these reference voltages to their actual values, one may determine the slope and offset of the actual ADC. This information may then be used to correct for differences between the actual and the desired values of offset and slope.

(a). Assume that we have a 14 bit A/D converter. Then the full scale voltage V_{max} should be converted to $2^{14} = 16384 = 0\text{x}4000$. The reference voltage $V_1 = 25\% V_{max}$ should be converted[2] to $2^{14} \times 0.25 = 0\text{x}1000 = 4096$ and the reference voltage $V_2 = 75\% V_{max}$ should be converted to $0\text{x}3000 = 12288$. Suppose instead that V_1 is converted to 0x100D and V_2 is converted to 0x2FE7. Find the actual gain and offset of the ADC transfer function. Express your answer as a straight line $y = mx + b$ where x is expressed as a fraction of V_{max}, and y is expressed as a fraction of 2^{14}, the 14-bit binary representation of V_{max}.

(b). Use the information from the previous item to find an equation that takes an actual converted value y_{raw}, expressed as a fraction of 2^{14}, and corrects for the gain and offset errors in the ADC, yielding a result $y_{corrected}$ that is also expressed as a fraction of 2^{14}. Your answer should have the form $y_{corrected} = \alpha y_{raw} + \beta$. You may assume that the required calculations are carried out in floating point (the application note AN2989 describes how finite wordlength may be accounted for, but you need not do so).

(c). Why is it not always necessary to calibrate an ADC? Give examples. HINT: Read the application note AN2989.

[2] Conversions between hex and decimal values may be performed with the Matlab commands `hex2dec` and `dec2hex`.