EECS 461, Fall 2009, Problem Set 5[1]

issued: Thursday October 15, 2009 due: Tuesday October 27, 2009

1. Consider the Simulink diagram in Figure 1, which we have previously used to model a virtual wall.

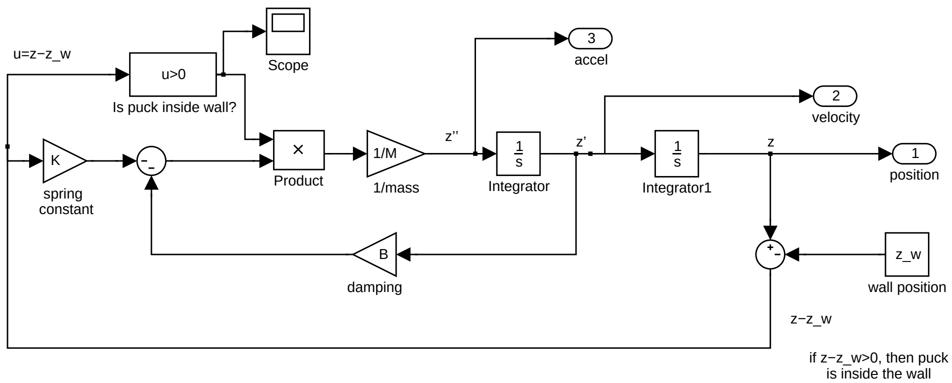

Figure 1: SIMULINK Model of Virtual Wall with Spring and Damping

An alternate way to simulate the virtual wall is by using Stateflow to model the fact that the puck is in either one of two discrete states: outside the wall, or inside the wall. This way of modelling the two discrete states may at first appear more complicated than the approach shown in Figure 1. Suppose, however, that the system we are trying to model and simulate has several discrete states. Then the simple trick used in Figure 1 will not work to distinguish between them. A simple example of a system with three discrete states is shown in Figure 2, which represents a one dimensional version of a Pong game. The virtual ball can be in one of three discrete states: in contact with the left paddle, in contact with the right paddle, or in contact with neither. If one were to model a two dimensional version of the Pong game, then the ball could be in one of several discrete states, depending on whether it was in contact with one of the paddles, or with the sides of the playing area, or in free flight. It is much more natural to model these situations using several discrete states.

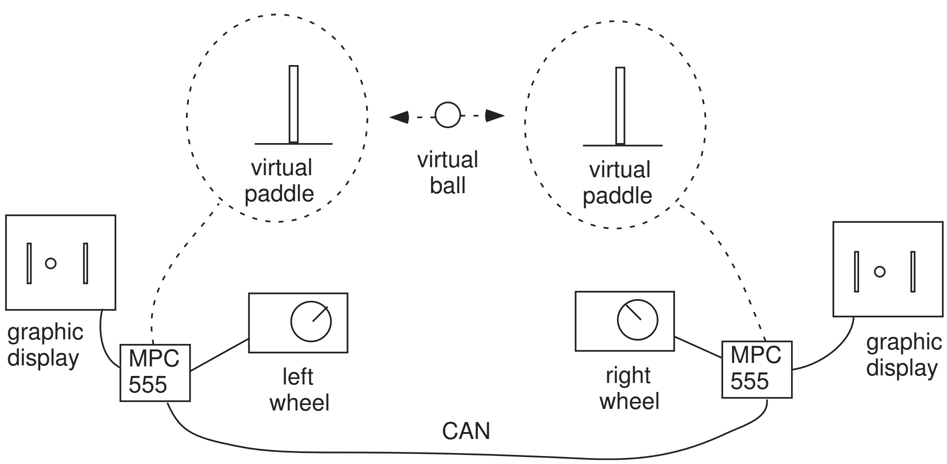

Figure 2: A virtual Pong game with three discrete states

In the remainder of this problem, we will develop a Stateflow model of the virtual wall, and then extend the model to a system with two virtual walls, and thus three discrete states.

[1]Revised October 15, 2009.

(a) Modify the Simulink model in Figure 1 to that shown in Figure 3, with the Stateflow diagram shown in Figure 4. (Assume that the puck has an initial position inside the wall.) Use the Matlab file "prob1_PS5.m" to compare the results of the two simulations. They should be very similar, but if you zoom on the plots in the fifth plot, you should see some slight differences. Why do these differences exist? Hand in a copy of the third and fifth plots obtained from the Matlab file. NOTE: In the Tools → Explore menu of the Statechart, the variables `zdot` and `Deltaz` should have scope "Input", `F` should have scope "Output", and `K`, `B`, and `Deltaz0` should have scope "Parameter". All variables should have data type "double".

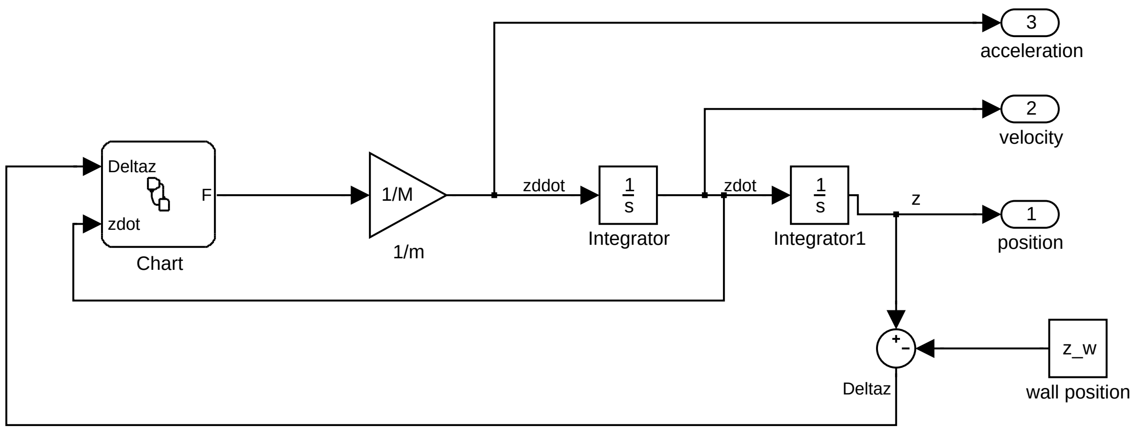

Figure 3: SIMULINK model of virtual wall with Stateflow used to model two discrete states

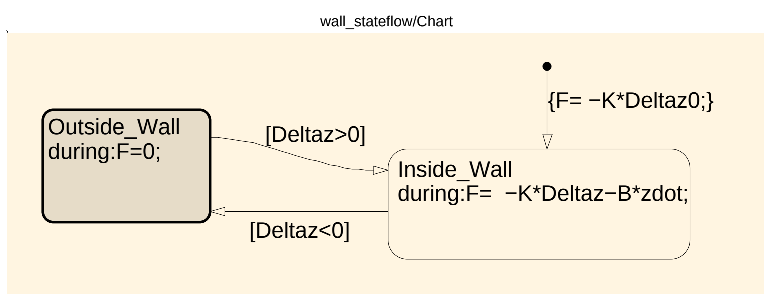

Figure 4: Details of Stateflow block

(b) Modify the Matlab, Simulink, and Stateflow files you used above to model what happens when the puck must move between *two* virtual walls, as depicted in Figure 5. When the puck is inside either wall, it will experience the restoring force of the appropriate spring and damper. When the puck is between the walls, it will experience zero force. It is assumed that the puck has initial condition inside the right wall. The Simulink file you will use should look like that in Figure 6. Hand in: a copy of the Stateflow block that goes with Figure 6 and a plot of the response of the puck for $B = 0$. (The latter response should look like that in Figure 7.)

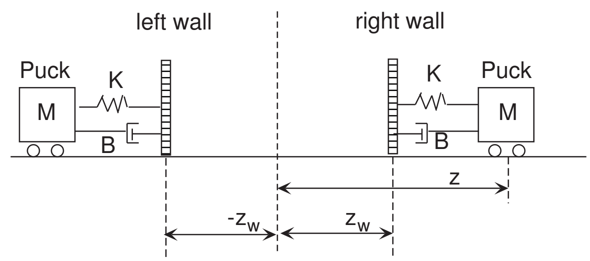

Figure 5: Two Virtual Walls

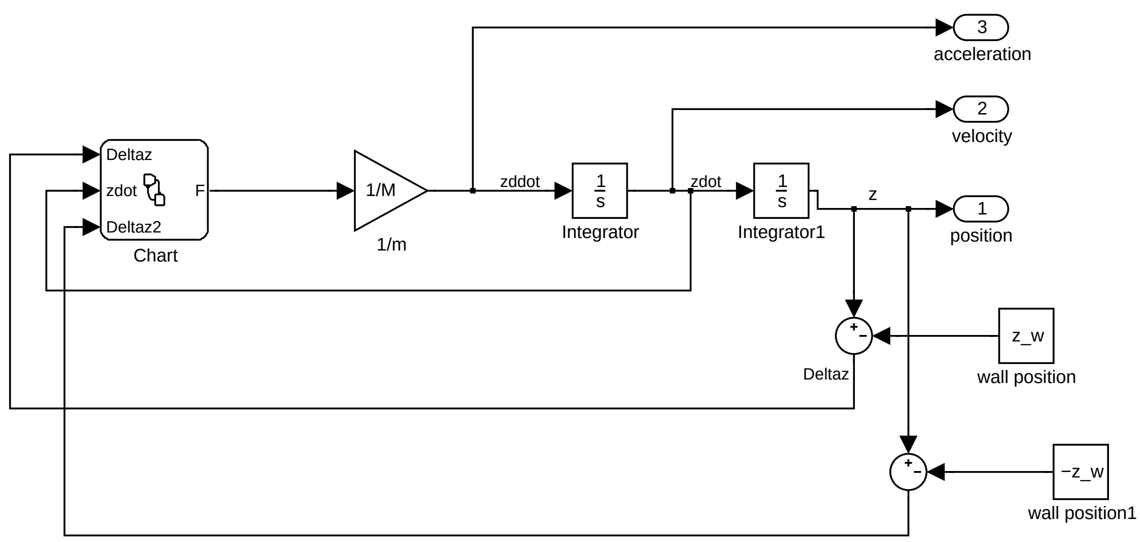

Figure 6: SIMULINK model of puck trapped between two virtual walls

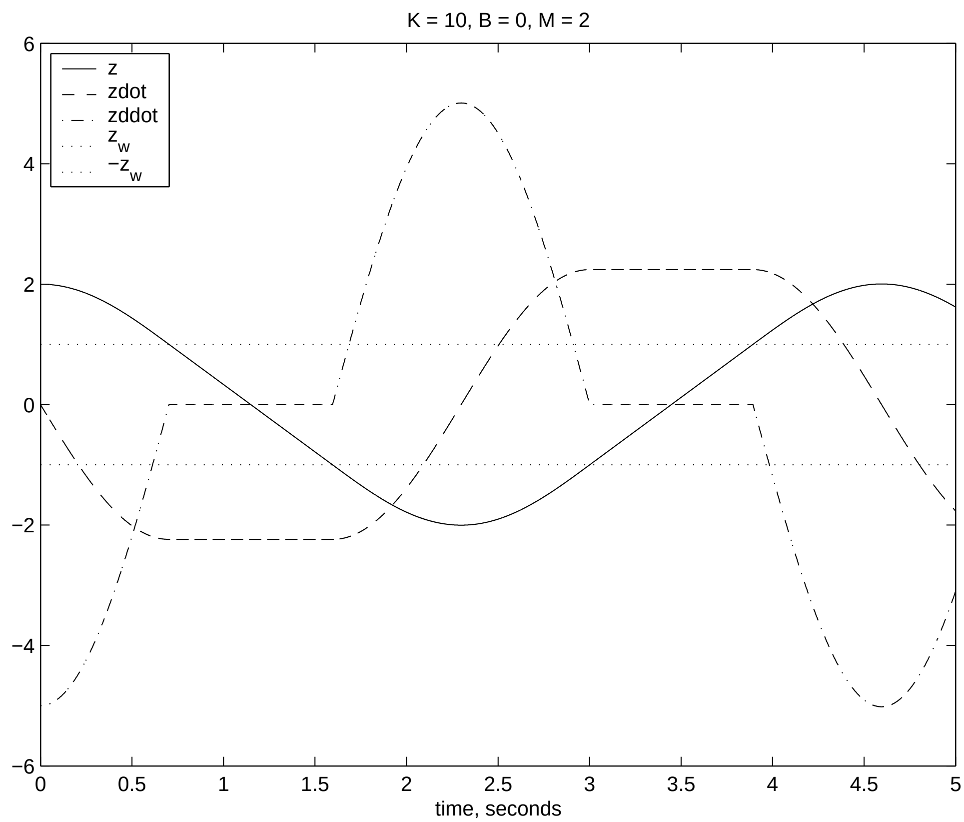

Figure 7: Response of puck to two walls

2. Consider the equations of motion of a virtual world consisting of a virtual inertia, J, attached to the haptic wheel by a torsional spring with constant k

$$\ddot{\theta}_w + \frac{k}{J}\theta_w = \frac{k}{J}\theta_z, \tag{1}$$

where θ_w and θ_z denote the angles of the virtual and haptic wheels, respectively. Suppose that initially both wheels are at their reference locations, $\theta_w = \theta_z = 0$, and that at time $t = t_0$ the haptic wheel is turned to an angle θ_{z0} and held constant thereafter.

(a) Verify that the position of the virtual wheel satisfies the equation

$$\theta_w(t) = \theta_{z0}\left(1 - \cos(\omega_n(t - t_0))\right), \quad t \geq t_0, \tag{2}$$

where $\omega_n = \sqrt{k/J}$ radians/second.

(b) Derive an expression for the torque acting on the virtual wheel as a function of time. What is the maximum magnitude of this torque?

3. State variable models can be applied in many situations. When modelling a mechanical system the state variables (or states) are generally chosen to be the position and velocity of each mass in the system. (If the system exhibits rotary motion, the states are chosen to be the angular position and angular velocity of each inertia.) If the system to be modelled is an electric circuit, such as the RLC circuit depicted in Figure 8, then the states may be chosen as the current through each inductor and the voltage across each capacitor

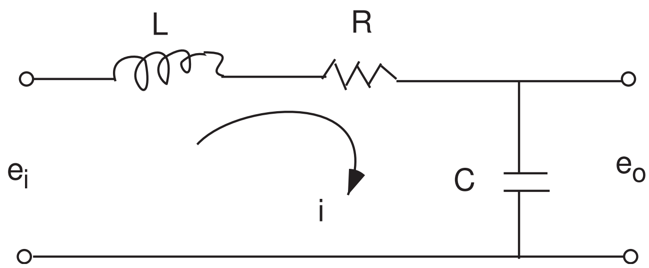

Figure 8: RLC circuit.

(a) Consider the RLC circuit shown in Figure 8. Kirchoff's laws state that the voltages e_o and e_i and the current i are related by the equations:

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = e_i \tag{3}$$

$$\frac{1}{C}\int i\,dt = e_o. \tag{4}$$

Find the transfer function from input voltage e_i to output voltage e_o. HINT: First find the transfer functions from e_i to i and from i to e_o.

(b) Find a second order differential equation relating input voltage e_i to output voltage e_o.

(c) Define state variables $x_1 = e_o$, $x_2 = \dot{e}_o$, input $u = e_i$, and output $y = e_o$. Find a state variable representation of the system in the form

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

(d) For what values of the parameters R, L, and C is the circuit an oscillator?

(e) Let parameter values be chosen so that the circuit is an oscillator with natural frequency $\omega_n = 20$ radians/second. Suppose that the circuit is simulated on a computer using Forward Euler integration with simulation step size T seconds. Where do the eigenvalues of the discrete system lie, as a function of T, for the specified natural frequency?

(f) How should we change the values of the circuit parameters if we wish its discrete approximation to be an oscillator?

4. Consider the feedback system in Figure 9, where $\alpha > 0$. Denote the Laplace transforms of the input $r(t)$ and the output $y(t)$ by $R(s)$ and $Y(s)$, respectively.

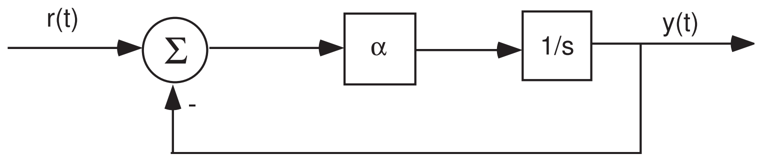

Figure 9: Feedback System with Analog Integrator

(a) Find the transfer function from $R(s)$ to $Y(s)$ and the differential equation that must be satisfied by $y(t)$. Is the system stable? If so, find the steady state response of $y(t)$ to a step input $r(t) = 1, t \geq 0$.

(b) Recall that the time constant of the system in Figure 9, denoted by τ, describes the rate at which the response to a constant input converges to its steady state value. Find an expression for τ in terms of α. Describe qualitatively how the value of α will affect the *transient* response of the system to a step input $r(t) = 1, t \geq 0$.

(c) What value of α will result in a time constant $\tau = 0.2$? With this value of τ, the response will converge to its final value at the same rate as certain function $f(t) \to 0$. What is $f(t)$?

(d) Suppose that the analog integrator $1/s$ is replaced by a discrete integrator $T/(z-1)$, as shown in Figure 10, where $T > 0$ denotes the sample period. Denote the input and output sequences by $\{r(kT)\}$ and $\{y(kT)\}$, and their z-transforms by $R(z)$ and $Y(z)$. Find the transfer function from $R(z)$ to $Y(z)$ and the difference equation that must be satisfied by $y(kT)$.

(e) Find all values of T for which the discrete system is stable. Express your answer in terms of the constant α. What values of the time constant require faster sampling?

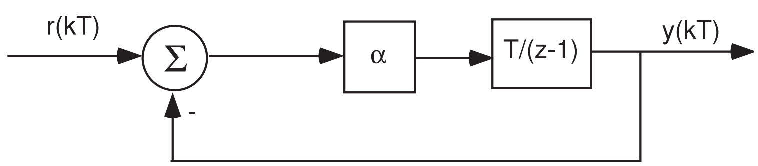

Figure 10: Feedback System with Discrete Integrator

(f) Suppose that the value of T is given and that we wish the discrete system to have a characteristic root at e^{-5T}. What value of α will achieve this? For small values of T, how does this value of α compare with that from part (c)?