

# Actuators

- The other side of the coin from sensors...
- Enable a microprocessor to modify the analog world.
- Examples:
  - speakers that transform an electrical signal into acoustic energy (sound)
  - remote control that produces an infrared signal to control stereo/TV operation
  - motors: used to transform electrical signals into mechanical motion
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- Actuator interfacing issues:
  - physical principles
  - interface electronics
  - power amplification
  - “advanced D/A conversion”: How to generate an analog waveform from a discrete sequence?

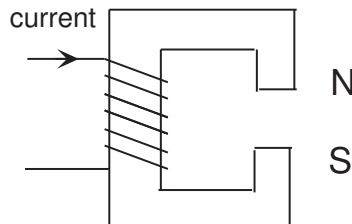
# Motors

- Used to transform electrical into mechanical energy using principles of electromagnetics
- Can also be used in reverse to convert mechanical to electrical energy
  - generator
  - tachometer
- Several types, all of which use electrical energy to turn a shaft
  - DC motors: shaft turns continuously, uses direct current
  - AC motors: shaft turns continuously, uses alternating current
  - stepper motors: shaft turns in discrete increments (steps)
- Many many different configurations and “subtypes” of motors
- Types of DC motors
  - brush
  - brushless
  - linear
- We shall study brush DC motors, because that is what we will use in the laboratory
- References are [4], [2], [1], [3], [6], [5]

# Electromagnetic Principles

Electromagnetic principles underlying motor operation:

- a flowing current produces a magnetic field whose strength depends on the current, nearby material, and geometry
  - used to make an electromagnet



- motors have either permanent magnets or electromagnets
- a current,  $I$ , flowing through a conductor of length  $L$  in a magnetic field,  $B$ , causes a force,  $F$ , to be exerted on the conductor:

$$F = k_1 B L I$$

where the constant  $k_1$  depends on geometry

- idea behind a motor: use this force to do some mechanical work
- a conductor of length  $L$  moving with speed  $S$  through a magnetic field  $B$  has a potential difference between its ends

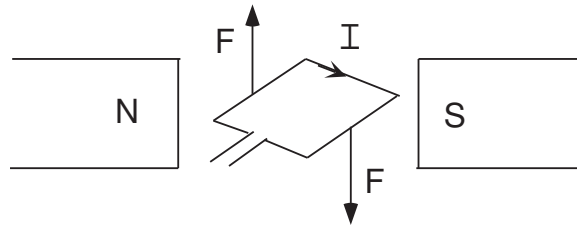
$$V = k_2 B L S$$

where the constant  $k_2$  depends on geometry

- idea behind a generator: use this potential difference to generate electrical power

## Simplistic DC Motor

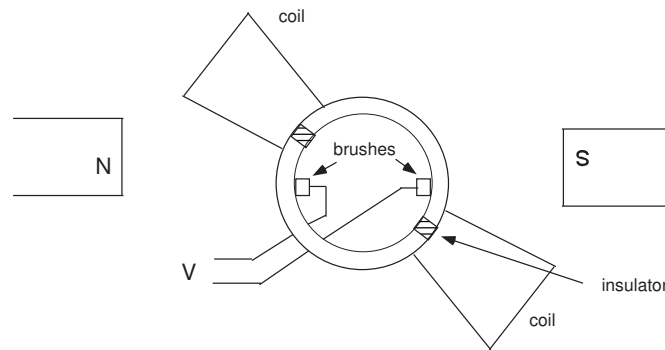
- A motor consists of a moving conductor with current flowing through it (the *rotor*), and a stationary permanent or electromagnet (the *stator*)
- consider a single loop of wire:



- combined forces yield a torque, or angular force, that rotates the wire loop
  - recall the “right hand rule” from physics
- Problems:
  - the force acting on the wire rotates it clockwise half the time, and counterclockwise half the time
  - if we want only CW rotation, we must turn off the current and let the rotor coast, during the time when the force is in the wrong direction
  - dead spot: there is one position where the force is zero

## Brushes, Armature, and Commutator

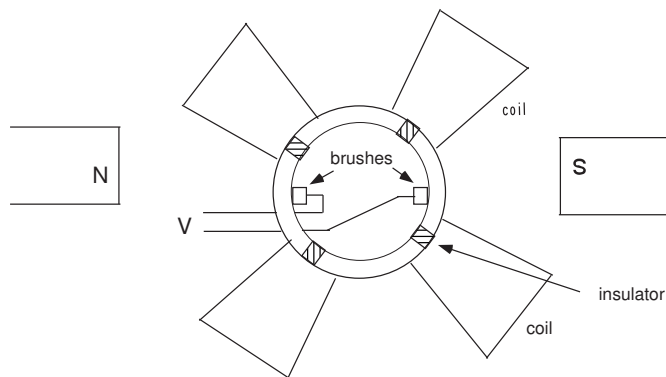
- Armature: the current carrying coil attached to the rotating shaft (rotor), which is divided into electrically isolated areas
- Commutator: uses electrical contacts (brushes) on the rotating shaft to switch the current back and forth
- Every time the brushes pass over the insulating areas, the direction of current flow through the coils changes, so that force is always in the same direction



- Problems:
  - still a dead spot, where no torque is produced.
  - torque varies greatly depending on geometry

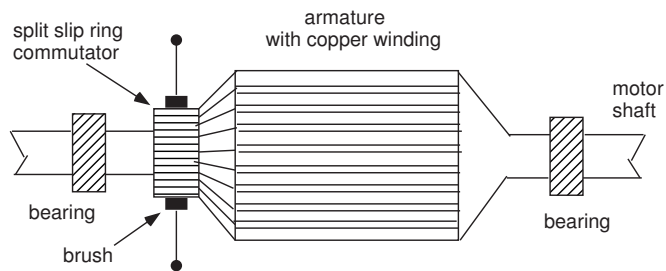
# Practical Motor

- Adding more coils and brushes removes dead spot and allows smoother torque production



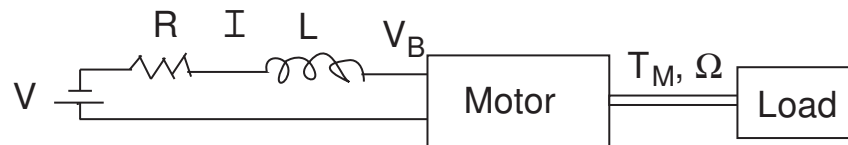
- disadvantages of brush DC motors
  - electrical noise
  - arcing through switch
  - wear

- More realistic diagram:



# Motor Equations

- Mechanical variables on one side of motor, electrical variables on the other:

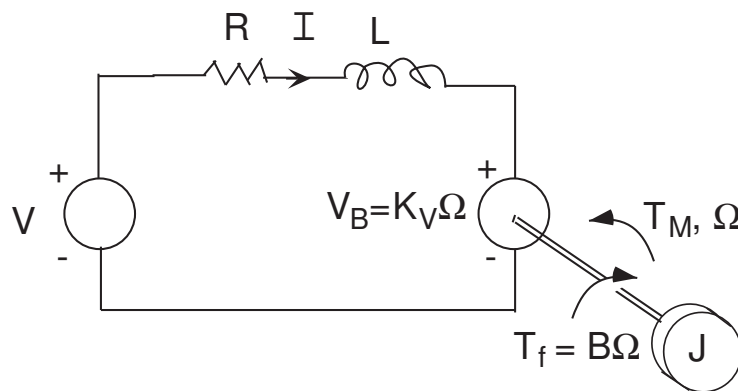


- Torque produced by motor as a result of current through armature:
$$T_M = K_M I$$
where  $T_M$  denotes motor torque,  $K_M$  is the torque constant, and  $I$  is the current through the armature.
- Voltage produced as a result of armature rotation (called the *back EMF*):
$$V_B = K_V \Omega$$
where  $V_B$  is the back emf,  $K_V$  is the emf constant, and  $\Omega$  is the rotational velocity
- Units:
  - $T_M$ : motor torque, Newton-meters
  - $I$ : current, Amps
  - $V_B$ : back emf, Volts
  - $\Omega$ : rotational velocity, radians/second
- In these units,  $K_M$  (N-m/A) =  $K_V$  (V/(rad/sec))

# Circuit Equivalent

- Notation:

- $J$ : inertia of shaft
- $T_f = B\Omega$ : friction torque
- $R$ : armature resistance
- $L$ : armature inductance (often neglected)



- Current: 
$$V - V_B = RI + L \frac{dI}{dt} \quad (1)$$

- Torque: 
$$T_M = K_M I \quad (2)$$

- Back EMF: 
$$V_B = K_V \Omega \quad (3)$$

- In steady state ( $\frac{dI}{dt} = 0$ ), substitute (2) into (1), rearrange, and apply (3):

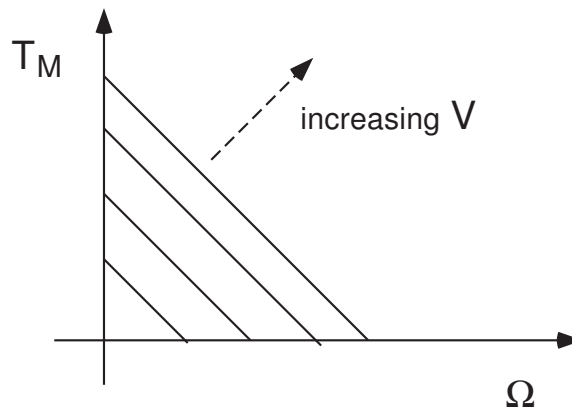
$$\begin{aligned} T_M &= \frac{K_M(V - V_B)}{R} \\ &= \frac{K_M(V - K_V \Omega)}{R} \end{aligned} \quad (4)$$

## Torque-Speed Curves

- For a fixed input voltage  $V$ , the torque  $T_M$  produced by the motor is inversely proportional to the rotational speed  $\Omega$ :

$$T_M + \left( \frac{K_M K_V}{R} \right) \Omega = \left( \frac{K_M}{R} \right) V \quad (5)$$

- Graphically:



- Maximum torque achieved when speed is zero:

$$T_M = \left( \frac{K_M}{R} \right) V$$

- Maximum speed achieved when torque is zero:

$$\Omega = \left( \frac{1}{K_V} \right) V$$

- Tradeoff between speed and torque should be familiar from riding a bicycle!

## Load Torque

- Recall Newton's law for forces acting on mass:

$$\sum \text{forces} = ma$$

where  $m$  is the mass and  $a$  is acceleration.

- Analogue for rotational motion is

$$\sum \text{torques} = J \frac{d\Omega}{dt}$$

where  $J$  is inertia, and  $\frac{d\Omega}{dt}$  is angular acceleration

- The shaft will experience
  - a torque  $T_M$  supplied by the motor,
  - a friction torque  $T_f = B\Omega$  proportional to speed
  - a load torque  $T_L$  due to the load attached to the shaft<sup>1</sup>
- Generally load and friction torques are opposed to motor torque:

$$T_M - T_f - T_L = J \frac{d\Omega}{dt}$$

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<sup>1</sup>We will assume load torque is constant, but it may also include a term proportional to angular velocity:  $T_L = T_1 + T_2\Omega$ .

## Speed under Load

- Torque equation

$$T_M - B\Omega - T_L = J \frac{d\Omega}{dt}$$

- In steady state,  $\frac{d\Omega}{dt} = 0$ , and applied torque equals load torque plus friction torque:

$$T_M = T_L + B\Omega \quad (6)$$

- Recall torque equation

$$T_M = \frac{K_M(V - K_V\Omega)}{R} \quad (7)$$

- Setting (7) equal to (6) and solving for  $\Omega$  shows that steady state speed and torque depend on the constant load  $T_L$ :

$$\Omega = \frac{K_M V - R T_L}{K_M K_V + B R} \quad (8)$$

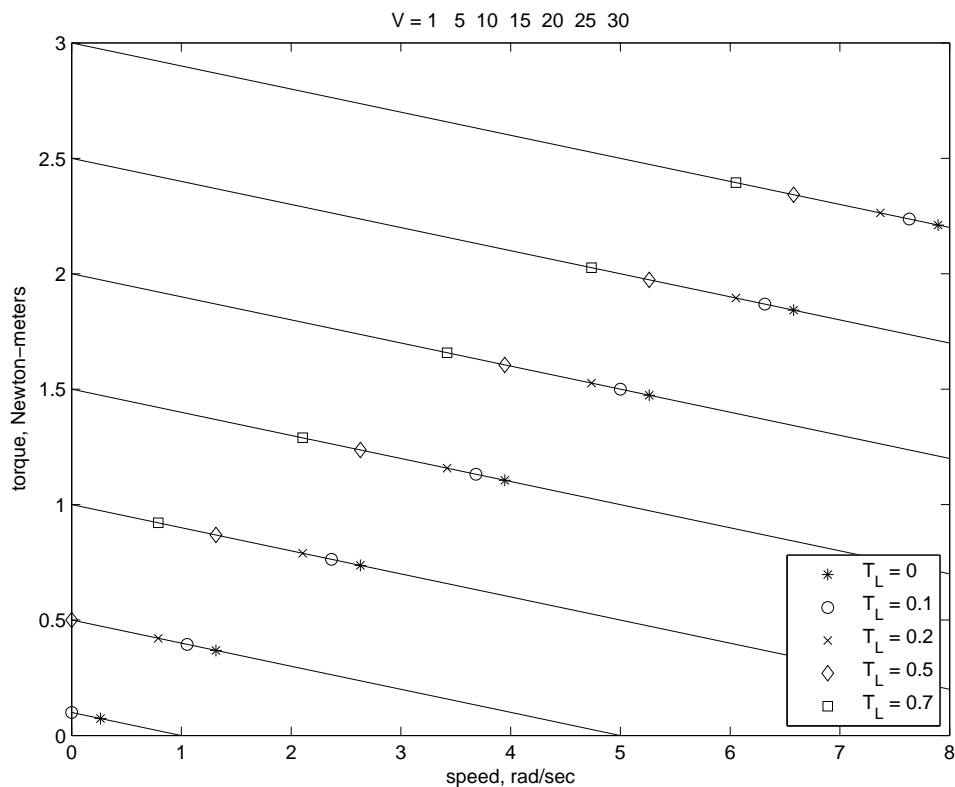
Substituting (5) yields

$$T_M = \frac{K_M(VB + K_V T_L)}{K_M K_V + B R} \quad (9)$$

- Generally, load torque will decrease the steady state speed
- Motor will also produce nonzero torque in steady state
- Note: (8) and (9) must still satisfy torque-speed relation (5)
- Location on a given torque/speed curve depends on the load torque

## Example

- Motor Parameters
  - $K_M = 1$  N-m/A
  - $K_V = 1$  V/(rad/sec)
  - $R = 10$  ohm
  - $L = 0.01$  H
  - $J = 0.1$  N-m/(rad/sec<sup>2</sup>)
  - $B = 0.28$  N-m/(rad/sec)
- Input voltage:  $V = [1 \ 5 \ 10 \ 15 \ 20 \ 25 \ 30]$
- Load torque:  $T_L = [0 \ 0.1 \ 0.2 \ 0.5 \ 0.7]$
- Torque-speed curves<sup>2</sup>



<sup>2</sup>MATLAB plot torque\_speed\_curves.m

## Motor as a Tachometer

- We think of applying electrical power to a motor to produce mechanical power.
- The physics works both ways: we can apply mechanical power to the motor shaft and the voltage generated (back emf) will be proportional to shaft speed:

$$V_B = K_V \Omega$$

⇒ we can use the motor as a tachometer.

- Issues:
  - brush noise
  - voltage constant drift

## References

- [1] D. Auslander and C. J. Kempf. *Mechatronics: Mechanical Systems Interfacing*. Prentice-Hall, 1996.
- [2] W. Bolton. *Mechatronics: Electronic Control Systems in Mechanical and Electrical Engineering, 2nd ed.* Longman, 1999.
- [3] C. W. deSilva. *Control Sensors and Actuators*. Prentice Hall, 1989.
- [4] G.F. Franklin, J.D. Powell, and A. Emami-Naeini. *Feedback Control of Dynamic Systems*. Addison-Wesley, Reading, MA, 3rd edition, 1994.
- [5] C. T. Kilian. *Modern Control Technology: Components and Systems*. West Publishing Co., Minneapolis/St. Paul, 1996.
- [6] B. C. Kuo. *Automatic Control Systems*. Prentice-Hall, 7th edition, 1995.