

EECS 477. Homework 1.

Due on Thursday 9/12/2002 before the class

You must show all work to receive credit!

Please read the statement below and sign your name; otherwise, your homework will not be graded. The text of the College of Engineering's Honor Code can be found at <http://honor.personal.engin.umich.edu/>

I hereby acknowledge that I understand the College of Engineering's Honor Code and have pledged to uphold it and abide by it.

Signature: _____

1. Summation (20 points)

Prove by mathematical induction that

$$\sum_{i=1}^n (2i-1)^2 = \frac{n(4n^2-1)}{3}.$$

2. Divisibility (20 points)

Prove by mathematical induction that $n^5 - n$ is divisible by 5 for all $n \geq 1$.

3. Quantifiers (10 points)

Is the statement below true or false? Show why.

$$(\exists p \in \mathbf{N})(\exists q \in \mathbf{N})[(p \text{ is prime}) \wedge (q \text{ is prime}) \wedge (pq + 6 \text{ is prime})]$$

4. Who's the killer? (30 points)

A horrendous crime was committed and Inspector Maigret is looking for the killer. His assistants tell him the following:

- If Francois was drunk then either Etienne is the killer or Francois is lying (or both).
- Etienne is the killer or Francois was not drunk and the crime happened after midnight.
- If the crime happened after midnight then either Etienne is the killer or Francois is lying (or both).

Consider the following statements:

- A Francois was drunk
- B Etienne is the killer
- C Francois is lying
- D the crime happened after midnight

What follows from the above evidence?

Inspector Maigret knows for sure that sober Francois never lies. What conclusion can he make?

(Model the situation in terms of mathematical notation from Chapter 1 and use it to solve the problem.)

5. Limits (20 points)

a) Find the limit

$$\lim_{n \rightarrow \infty} \frac{5^{n+2}}{10^n}.$$

b) Find the limit

$$\lim_{n \rightarrow \infty} \frac{n}{2^n}.$$