Final overview

EECS 477
Lecture 24, 12/10/2002

Asymptotic notation

- $O(n)$, $\Omega(n)$, $\Theta(n)$
  - Know them by heart
- Average, best, and worst performance
- Elementary operations
- Lower bounds and upper bounds
- Problems, algorithms, instances
- Barometers, simple control structures
Recurrences

- Recursion, divide and conquer
- Inhomogeneous recurrence
  - General form of solution
  - Taking care of the RHS and initial conditions
  - Multiple roots of characteristic equation
  - Transformations
    - Change of variable
    - Change of range

Master Theorem

- \( T(n) = a \ T(n/b) + f(n) \)
- Three cases
  - \( f(n) = O(n^{\log_b a - \epsilon}) \) then \( T(n) = \Theta(n^{\log_b a}) \)
  - \( f(n) = \Theta(n^{\log_b a - \epsilon}) \) then \( T(n) = \Theta(n^{\log_b a \log n}) \)
  - \( f(n) = \Omega(n^{\log_b a + \epsilon}) \) (and if \( af(n/b) \leq cf(n) \) for some constant \( c < 1 \)) then \( T(n) = \Theta(f(n)) \)
- If `<=` then can claim \( O \) result
- There are gaps in the theorem
Data structures

- Heaps (various)
- Disjoint sets
- Trees
- Hash tables
- Vectors and other simple ones

Graphs

- Complete, connected, strongly connected, articulation points, spanning trees, paths, cycles, DAGs
- Two simple traversals
  - DFS, BFS: What is runtime asymptotics?
- Two representations
- Two parameters
Graphs with edge weights

- MST: $O( E \log V )$
  - Kruskal: shortest edges first
  - Prim: grow a tree
- Dijkstra: $O( (A+N) \log N )$ or $O(N^2)$
  - Shortest paths from a vertex to everybody else
- Asymptotics
  - Two parameters!

Knapsack: variants

- Greedy
  - Optimal for breakable objects
- Dynamic programming
  - Types of objects
- NP-complete
  - Non-breakable objects
- Approximate algorithm
  - Modifies the greedy
Greedy algorithms

- Scheduling
  - Min time in the system
  - Unit time jobs with deadlines
- Making change
  - Optimal for some coinage
- Often
  - Basis for heuristic algorithms
    - Coloring, metric TSP

Divide and conquer

- Long integer arithmetic
  - $a*b$ MT, different lengths
- Strassen
  - Matrix multiplication
- Median in linear time
- Mergesort
Dynamic programming

- Shortest paths from everybody to everybody
  - Floyd's algorithm
- Chained matrix multiplication
- Table
  - Making change
  - Knapsack
- Memory functions

Traveling salesman: variants

- Trivial n! permutations
- Dynamic programming: $\Theta(n^2 2^n)$
  - Builds $\text{Opt}[S; x] = \text{length of the cheapest path starting in city } 1 \text{ visiting all the cities in } S\setminus\{x\} \text{ and stopping in city } x$
- Branch and bound
  - Exclude/include edges one by one
    - That gives constraints
- Metric TSP
Backtracking, Branch&Bound

- Games
- Eight queens
- Assignment
  - Jobs to workers
- TSP
- Minimax

Complexity

- Lower bounds
  - Information theoretical
    - Game of K questions
    - Average leaves depth in a tree
      - Sorting complexity
  - Adversary arguments
    - Finding maximum
    - Finding median
    - Graph connectivity
P and NP

- Optimization and Decision problems
  - Classes P and NP
- Polynomial reduction
- NP-completeness
  - SAT-CNF, SAT-3-CNF
  - TSPD, HAMD, HAM
- NP-hardness

Approximate algorithms

- Knapsack
  - Modified from greedy
- Metric TSP
  - MST

Book sections
- 3, 4, 5
- 6.1-6.5, 6.6.1
- 7.1-7.6
- 8, 9
- 12.2, 12.3, 12.5
- 13.1-13.2