EECS 477 - Sample Final Exam

Name -

UMich ID # -

DO NOT OPEN THE EXAM BOOKLET UNTIL YOU ARE INSTRUCTED TO BEGIN!

Honor Code: I have neither given nor received any help on this exam.

Signature: ____

You must work by yourself. This is a closed book exam. You are allowed to use one 4" x 6" index card with anything you like written on it. No other material may be consulted during the exam. You may not use any scratch paper other than what is provided in the exam booklet.

Please read all directions and questions carefully.

You have 1 hour and 50 minutes to complete this exam.

Good luck.

1. **Algorithmic Complexity** (20 points)

Directions: Circle the best answer from among the choices listed for each of the following questions.

a) What is the running time of the greedy algorithm that finds the optimal solution to the variation of the Knapsack problem that allows fractions of objects? Let n be the number of objects.

$O(n^2)$ $O(n^3)$ $O(n \log n)$

$O(2^n)$ none of these

b) What is the running time of Floyd's algorithm for finding all-pairs shortest paths?

$\Theta(|V|^2)$ $\Theta(|E|^2)$ $\Theta(|V| + |E|)$

$\Theta(|V|^3)$ $\Theta(|V||E|)$

c) What is the running time for Strassen's method for matrix multiplication?

$O(n \log n)$ $O(n^{2.81})$ $O(n^3)$

$O(n^2 \log n)$ $O(n^2)$

d) What is the running time of topological sort?

$\Theta(|V|^2)$ $\Theta(|E|^2)$ $\Theta(|E| \log |V|)$

$\Theta(|V| \log |E|)$ $\Theta(|V| + |E|)$

2. **Fill in the blanks** (20 points)

Strassen's method for matrix multiplication employs this algorithmic strategy: ____________.

Topological sort is implemented using ____________ search.

____________ is a bottom-up algorithmic technique whereas divide-and-conquer is a top-down approach.

In Computer Science, "NP" stands for ____________ ____________ .

To find the shortest hop-distance between two vertices in a given graph in linear time, one would use the ____________ algorithm.

Union-find (aka disjoint-sets) data structures are used in a classical algorithm for the ____________ problem.

Topological sorting only applies to ____________ ____________ graphs.

Dynamic programming can be applied to problems that exhibit the principle of ____________ .

A disconnected graph does not have a spanning tree, but has a spanning ____________ . The ____________ algorithm applied to a disconnected graph without modifications will be able to build a spanning ____________, but the ____________ algorithm will not.

The complexity class NP is the class of languages that can be ____________ by a polynomial-time algorithm.

3. **Supply Chain Management** (20 points)

A car company has several different factories which supply each other with parts. There is a cost per ton of parts (or just cost) associated with shipping parts from factory A to factory B, and it is not necessarily the same as the cost of shipping parts from factory B to factory A.

Parts do not have to be shipped directly from the source factory to the destination factory; they may be shipped via stops at one or more factories in between. Also, there may not necessarily be a direct shipping link between every pair of factories. Given two factories A and B, shipments can be made in at most one direction; this means that there is either a shipping link from A to B, a shipping link from B to A, or no link at all.

The company has hired you to build a table that for every pair of factories shows the minimum cost of shipping. Given that there are a total of n factories, use C/C++ style pseudocode to describe an algorithm that solves this problem. You should try to come up with as fast an algorithm as possible. Show an analysis of your algorithm's running time and memory complexity in terms of n.

You can assume that you are given the shipping costs for each pair of factories that have a direct shipping link between them in an n x n table. The table contains an X for factories that don't have a direct shipping link between them. You are allowed to assume that the supply chain is *sound*, which means that it has no cyclic dependencies.

Your algorithm is not allowed to make calls to any algorithms discussed in class or in the book, but you may find it helpful to use them as a framework for your design.

4. **Company Party** (20 points)

Professor McKenzie is consulting for the president of Alpha-Beta Corporation, who is planning a company party. The company has a hierarchical structure; that is, the supervisor relation forms a tree rooted at the president. The personnel office has ranked each employee with a conviviality rating, which is a real number. In order to make the party fun for all attendees, the president does not want both an employee and his or her immediate supervisor to attend.

a) Use C/C++ style pseudocode to describe an algorithm to make up an optimal guest list. The goal should be to maximize the sum of the conviviality ratings of the guests subject to the immediate supervisor constraint. Analyze the running time of your algorithm.

Your algorithm should run in linear time in terms of n, where n is the number of employees in the company. Your algorithm must be correct and will be graded based on its runtime and the accuracy of your complexity analysis.

Your algorithm is not allowed to make calls to any algorithms discussed in class or in the book, but you may find it helpful to use them as a framework for your design.

b) How can the professor ensure that the president gets invited to his or her own party?

5. **Complexity Theory** (20 points)

a) Define CNF-SAT, 2-CNF-SAT, and 3-CNF-SAT

b) Consider an instance of Boolean Satisfiability with V variables, C clauses, and L literals. Given a truth assignment for the Boolean formula, what are the best-case and worst-case running times for verifying whether or not the assignment satisfies the formula? What are the best-case and worst-case memory complexities?

c) Show that 2-CNF-SAT $\in$ P

6. **BONUS** (10 points)

NO PARTIAL CREDIT!

In the context of problem #3, you can no longer make the assumption that the supply chain is *sound*. Explain how your algorithm from problem #3 could be used to detect the presence of cyclic shipping routes.