EECS483 D13: SSA Example

Chun-Hung Hsiao

April 12, 2013
Announcements

• Homework 5 on CTools
  – Due on 4/22
Static Single Assignment Form

• Each variable is given a unique name when it is assigned to a new value

• All of the uses of this assignment are renamed accordingly

• Phi nodes: a special multiplexer that choose a value from its arguments
SSA Conversion (1/2)
SSA Conversion (2/2)

• Dominator analysis
  – Find the dominator frontier set DF(BB) for each basic block BB

• Phi node insertion
  – If variable x is defined in BB, then a Phi node of x is needed in each basic block in DF(BB)

• Variable renaming
  – Rename variables in each assignment (including Phi node) and all their uses
Dominator Analysis (1/2)

• X dominates Y if every path from entry to Y contains X
  – X dominates X itself

• Z is a dominance frontier of X if X dominates a predecessor Y of Z but not Z
  – The first BB that is not dominated by X

• If variable a is defined in X
  – Uses of a in Y refer to the definition in X
  – Uses of a in Z don’t necessary refer to the definition in X
    • Need a Phi node for a!
Dominator Analysis (2/2)

• Dom(X) = Intersection(Dom(predecessors of X))

• Compute dominators
  – Initialization
    • Dom(Entry) = {Entry}
    • Dom(X) = {all nodes} for all other X
  – While(change):
    • Update Dom(X) for each X

• Compute dominance frontiers
  – for each Z
    • for each predecessor Y of Z
      – for each X in Dom(Y) - Dom(Z)
        » Put Z into DF(X)
Dominator Analysis: Example

This example comes from Prof. Mahlke’s EECS583 slides.
Dominator Analysis: Example

<table>
<thead>
<tr>
<th>BB</th>
<th>Dom</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0, 1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0, 1, 2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0, 1, 3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0, 1, 3, 4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0, 1, 3, 5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0, 1, 3, 6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0, 1, 7</td>
<td></td>
</tr>
</tbody>
</table>
Dominator Analysis: Example

\begin{itemize}
\item BB0
\item BB1
\item BB2
\item BB3
\item BB4
\item BB5
\item BB6
\item BB7
\end{itemize}

\begin{itemize}
\item a =
\item b =
\item c =
\item d =
\item a =
\item d =
\item b =
\item i =
\end{itemize}

\begin{tabular}{|c|c|c|}
\hline
BB & Dom & DF \\
\hline
0 & 0 & - \\
1 & 0, 1 & - \\
2 & 0, 1, 2 & 7 \\
3 & 0, 1, 3 & 7 \\
4 & 0, 1, 3, 4 & 6 \\
5 & 0, 1, 3, 5 & 6 \\
6 & 0, 1, 3, 6 & 7 \\
7 & 0, 1, 7 & 1 \\
\hline
\end{tabular}
Phi Node Insertion

- Liveness analysis
  - $\text{IN}(\text{BB})$: variables used in BB but defined elsewhere
  - $\text{KILL}(\text{BB})$: variables defined in BB

- Algorithm
  - for each variable $v$ in $\text{IN}(\text{BB})$ for some BB
    - $\text{Def}(v) = \{\text{BB} : v \in \text{KILL}(\text{BB})\}$
    - for each $\text{BB} \in \text{Def}(v)$
      - Insert a Phi node for $a$ in $\text{DF}(\text{BB})$
      - Add $\text{BB}$ into $\text{Def}(v)$
Phi Node Insertion: Example

a = c =  
b =  
c = d =  
i =  

a = Phi(a,a)  
b = Phi(b,b)  
c = Phi(c,c)  
d = Phi(d,d)  
i = Phi(i,i)  

variable | Def  
--- | ---  
a  
b  
c  
d  
i  

BB | Dom | DF  
--- | --- | ---  
0 | 0 | -  
1 | 0, 1 | -  
2 | 0, 1, 2 | 7  
3 | 0, 1, 3 | 7  
4 | 0, 1, 3, 4 | 6  
5 | 0, 1, 3, 5 | 6  
6 | 0, 1, 3, 6 | 7  
7 | 0, 1, 7 | 1  

Thursday, April 18, 2013
Phi Node Insertion: Example

BB0

BB1

BB2

BB3

BB4

BB5

BB6

BB7

<table>
<thead>
<tr>
<th>BB</th>
<th>Dom</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0, 1</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0, 1, 2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>0, 1, 3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>0, 1, 3, 4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>0, 1, 3, 5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>0, 1, 3, 6</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>0, 1, 7</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>variable</th>
<th>Def</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0, 1, 3</td>
</tr>
<tr>
<td>b</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td></td>
</tr>
<tr>
<td>i</td>
<td></td>
</tr>
</tbody>
</table>
Phi Node Insertion: Example

\[\text{a} = \Phi(\text{a}, \text{a})\]
\[\text{b} = \Phi(\text{b}, \text{b})\]
\[\text{c} = \Phi(\text{c}, \text{c})\]
\[\text{d} = \Phi(\text{d}, \text{d})\]
\[\text{i} = \Phi(\text{i}, \text{i})\]
Phi Node Insertion: Example

\[
\begin{align*}
BB0: & \quad a = \text{Phi}(a,a) \\
BB1: & \quad a = c = \\
BB2: & \quad b = c = d = \\
BB3: & \quad a = d = \\
BB4: & \quad d = \\
BB5: & \quad c = \\
BB6: & \quad b = \\
BB7: & \quad i = a = \text{Phi}(a,a)
\end{align*}
\]

<table>
<thead>
<tr>
<th>BB</th>
<th>Dom</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0, 1</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0, 1, 2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>0, 1, 3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>0, 1, 3, 4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>0, 1, 3, 5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>0, 1, 3, 6</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>0, 1, 7</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>variable</th>
<th>Def</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0, 1, 3, 7</td>
</tr>
<tr>
<td>b</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td></td>
</tr>
<tr>
<td>i</td>
<td></td>
</tr>
</tbody>
</table>

Thursday, April 18, 2013
Phi Node Insertion: Example

**Variable Definitions and Phi Node Insertion**

**Domino Nodes**:
- **BB0**: \(a = \Phi(a, a)\)
- **BB1**: \(a = \Phi(a, a)\)
- **BB2**: \(b = \Phi(b, b)\)
- **BB3**: \(c = \Phi(c, c)\)
- **BB4**: \(d = \Phi(d, d)\)
- **BB5**: \(c = \Phi(c, c)\)
- **BB6**: \(b = \Phi(b, b)\)
- **BB7**: \(i = \Phi(i, i)\)

**Domino Table**:

<table>
<thead>
<tr>
<th>BB</th>
<th>Dom</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0, 1</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0, 1, 2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>0, 1, 3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>0, 1, 3, 4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>0, 1, 3, 5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>0, 1, 3, 6</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>0, 1, 7</td>
<td>1</td>
</tr>
</tbody>
</table>

**Variable Definitions**:

- \(a\) is defined in \(0, 1, 3, 7\)
- \(b\) is defined in \(0, 2, 6\)
- \(c\) is defined in \(0, 1, 2, 5\)
- \(d\) is defined in \(0, 1, 3, 6\)
- \(i\) is defined in \(BB7\)

---

Thursday, April 18, 2013
Phi Node Insertion: Example

\[ a = \text{Phi}(a, a) \]

\[ b = \text{Phi}(b, b) \]

\[ c = \text{Phi}(c, c) \]

\[ d = \text{Phi}(d, d) \]

\[ i = \text{Phi}(i, i) \]

**BB** | **Dom** | **DF**
---|---|---
0 | 0 | -
1 | 0, 1 | -
2 | 0, 1, 2 | 7
3 | 0, 1, 3 | 7
4 | 0, 1, 3, 4 | 6
5 | 0, 1, 3, 5 | 6
6 | 0, 1, 3, 6 | 7
7 | 0, 1, 7 | 1

**variable** | **Def**
---|---
\( a \) | 0, 1, 3, 7
\( b \) | 0, 2, 6, 7
\( c \) |
\( d \) |
\( i \) |
Phi Node Insertion: Example

\[ a = \Phi(a, a) \]
\[ b = \Phi(b, b) \]

\[ a = \] Def
\[ b = \] Def
\[ c = \] Def
\[ d = \] Def

\[ BB \] Dom DF
\[ 0 \] 0 -
\[ 1 \] 0, 1 -
\[ 2 \] 0, 1, 2 7
\[ 3 \] 0, 1, 3 7
\[ 4 \] 0, 1, 3, 4 6
\[ 5 \] 0, 1, 3, 5 6
\[ 6 \] 0, 1, 3, 6 7
\[ 7 \] 0, 1, 7 1

variable Def
\[ a \] 0, 1, 3, 7
\[ b \] 0, 2, 6, 7, 1
\[ c \] 
\[ d \] 
\[ i \]
Phi Node Insertion: Example

BB0

\( a = \)
\( b = \)
\( c = \)
\( i = \)

BB1

\( a = \)
\( c = \)

BB2

\( b = \)
\( c = \)
\( d = \)

BB3

\( a = \)
\( d = \)

BB4

\( d = \)

BB5

\( c = \)

BB6

\( b = \)

BB7

\( i = \)

\( a = \text{Phi}(a,a) \)
\( b = \text{Phi}(b,b) \)

\( a = \text{Phi}(a,a) \)
\( b = \text{Phi}(b,b) \)

<table>
<thead>
<tr>
<th>BB</th>
<th>Dom</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0,1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0,1,2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>0,1,3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>0,1,3,4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>0,1,3,5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>0,1,3,6</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>0,1,7</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>variable</th>
<th>Def</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0,1,3,7</td>
</tr>
<tr>
<td>b</td>
<td>0,2,6,7,1</td>
</tr>
<tr>
<td>c</td>
<td>0,1,2,5</td>
</tr>
<tr>
<td>d</td>
<td></td>
</tr>
<tr>
<td>i</td>
<td></td>
</tr>
</tbody>
</table>
Phi Node Insertion: Example

\[
\begin{align*}
a &= \text{Phi}(a,a) \\
b &= \text{Phi}(b,b) \\
c &= \text{Phi}(c,c) \\
d &= \text{Phi}(d,d) \\
i &= \text{Phi}(i,i)
\end{align*}
\]

<table>
<thead>
<tr>
<th>BB</th>
<th>Dom</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0, 1</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0, 1, 2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>0, 1, 3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>0, 1, 3, 4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>0, 1, 3, 5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>0, 1, 3, 6</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>0, 1, 7</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>variable</th>
<th>Def</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0, 1, 3, 7</td>
</tr>
<tr>
<td>b</td>
<td>0, 2, 6, 7, 1</td>
</tr>
<tr>
<td>c</td>
<td>0, 1, 2, 5, 7, 6</td>
</tr>
<tr>
<td>d</td>
<td></td>
</tr>
<tr>
<td>i</td>
<td></td>
</tr>
</tbody>
</table>
Phi Node Insertion: Example

**Table: Dom and DF**

<table>
<thead>
<tr>
<th>BB</th>
<th>Dom</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0, 1</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0, 1, 2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>0, 1, 3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>0, 1, 3, 4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>0, 1, 3, 5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>0, 1, 3, 6</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>0, 1, 7</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table: Def**

<table>
<thead>
<tr>
<th>variable</th>
<th>Def</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0, 1, 3, 7</td>
</tr>
<tr>
<td>b</td>
<td>0, 2, 6, 7, 1</td>
</tr>
<tr>
<td>c</td>
<td>0, 1, 2, 5, 7, 6</td>
</tr>
<tr>
<td>d</td>
<td></td>
</tr>
<tr>
<td>i</td>
<td></td>
</tr>
</tbody>
</table>
Phi Node Insertion: Example

BB0

a =
b =
c =
i =

a = Phi(a,a)
b = Phi(b,b)
c = Phi(c,c)

BB1

a =
c =

BB2

b =
c =
d =

BB3

a =
d =

BB4

d =

BB5

c =

BB6

b =

c = Phi(c,c)

BB7

i =

a = Phi(a,a)
b = Phi(b,b)
c = Phi(c,c)

<table>
<thead>
<tr>
<th>BB</th>
<th>Dom</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0, 1</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0, 1, 2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>0, 1, 3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>0, 1, 3, 4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>0, 1, 3, 5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>0, 1, 3, 6</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>0, 1, 7</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>variable</th>
<th>Def</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0, 1, 3, 7</td>
</tr>
<tr>
<td>b</td>
<td>0, 2, 6, 7, 1</td>
</tr>
<tr>
<td>c</td>
<td>0, 1, 2, 5, 7, 6</td>
</tr>
<tr>
<td>d</td>
<td>2, 3, 4</td>
</tr>
<tr>
<td>i</td>
<td></td>
</tr>
</tbody>
</table>
Phi Node Insertion: Example

**Example**

```
a = Phi(a, a)
b = Phi(b, b)
c = Phi(c, c)
d = Phi(d, d)
i = Phi(i, i)
```

**Variable Definitions**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Def</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0, 1, 3, 7</td>
</tr>
<tr>
<td>b</td>
<td>0, 2, 6, 7, 1</td>
</tr>
<tr>
<td>c</td>
<td>0, 1, 2, 5, 7, 6</td>
</tr>
<tr>
<td>d</td>
<td>2, 3, 4, 7, 6</td>
</tr>
<tr>
<td>i</td>
<td></td>
</tr>
</tbody>
</table>

**Dominance Frontiers**

<table>
<thead>
<tr>
<th>BB</th>
<th>Dom</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0, 1</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0, 1, 2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>0, 1, 3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>0, 1, 3, 4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>0, 1, 3, 5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>0, 1, 3, 6</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>0, 1, 7</td>
<td>1</td>
</tr>
</tbody>
</table>

**Control Flow**

- BB0: a = Phi(a, a), b = Phi(b, b), c = Phi(c, c)
- BB1: a = c =
- BB2: b = c = d =
- BB3: a = d =
- BB4: d =
- BB5: c =
- BB6: b = c = Phi(c, c), d = Phi(d, d)
- BB7: i = a = Phi(a, a), b = Phi(b, b), c = Phi(c, c), d = Phi(d, d)
Phi Node Insertion: Example

BB0
- \( a = \)
- \( b = \)
- \( c = \)
- \( i = \)

BB1
- \( a = \)
- \( c = \)

BB2
- \( b = \)
- \( c = \)
- \( d = \)

BB3
- \( a = \)
- \( d = \)

BB4
- \( d = \)

BB5
- \( c = \)

BB6
- \( b = \)

BB7
- \( i = \)

\( a = \Phi(a,a) \)
\( b = \Phi(b,b) \)
\( c = \Phi(c,c) \)
\( d = \Phi(d,d) \)

<table>
<thead>
<tr>
<th>BB</th>
<th>Dom</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0, 1</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0, 1, 2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>0, 1, 3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>0, 1, 3, 4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>0, 1, 3, 5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>0, 1, 3, 6</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>0, 1, 7</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>variable</th>
<th>Def</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0, 1, 3, 7</td>
</tr>
<tr>
<td>b</td>
<td>0, 2, 6, 7, 1</td>
</tr>
<tr>
<td>c</td>
<td>0, 1, 2, 5, 7, 6</td>
</tr>
<tr>
<td>d</td>
<td>2, 3, 4, 7, 6, 1</td>
</tr>
<tr>
<td>i</td>
<td></td>
</tr>
</tbody>
</table>

Thursday, April 18, 2013
Phi Node Insertion: Example

Phi Node Insertion - Example

\[ a = \Phi(a, a) \]
\[ b = \Phi(b, b) \]
\[ c = \Phi(c, c) \]
\[ d = \Phi(d, d) \]

**Table:**

<table>
<thead>
<tr>
<th>BB</th>
<th>Dom</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0, 1</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0, 1, 2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>0, 1, 3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>0, 1, 3, 4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>0, 1, 3, 5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>0, 1, 3, 6</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>0, 1, 7</td>
<td>1</td>
</tr>
</tbody>
</table>

**Variable Definitions:**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Def</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0, 1, 3, 7</td>
</tr>
<tr>
<td>b</td>
<td>0, 2, 6, 7, 1</td>
</tr>
<tr>
<td>c</td>
<td>0, 1, 2, 5, 7, 6</td>
</tr>
<tr>
<td>d</td>
<td>2, 3, 4, 7, 6, 1</td>
</tr>
<tr>
<td>i</td>
<td>7</td>
</tr>
</tbody>
</table>

Thursday, April 18, 2013
Phi Node Insertion: Example

a = \Phi(a, a)
b = \Phi(b, b)
c = \Phi(c, c)
d = \Phi(d, d)
i = \Phi(i, i)

BB  | Dom     | DF
---|---------|---
0  | 0       | -
1  | 0, 1    | -
2  | 0, 1, 2 | 7
3  | 0, 1, 3 | 7
4  | 0, 1, 3, 4 | 6
5  | 0, 1, 3, 5 | 6
6  | 0, 1, 3, 6 | 7
7  | 0, 1, 7 | 1

variable | Def
---------|---
a       | 0, 1, 3, 7
b       | 0, 2, 6, 7, 1
c       | 0, 1, 2, 5, 7, 6
d       | 2, 3, 4, 7, 6, 1
i       | 7, 1
Variable Renaming (1/3)

- Constructing the dominator tree
  - The parent of a basic block is its immediate dominator

- For each variable, maintain the following data structures
  - A counter for creating new names
  - A stack to keep track of currently available names for this variable
    - The top of the stack is the name defined in its nearest dominators
Variable Renaming (2/3)

- Process each basic block in preorder of the dominator tree
  - Rewrite each instruction (including the Phi nodes) in forward order
    - For each use, replace the name with the latest name at the top of the stack
    - For each def, generate a new name
      - New name = original name + counter
      - Increment the counter by 1
      - Push the new name into the stack
  - Propagate the new names to the Phi nodes of its successors
  - Recursively process its children
  - Pop names generated in this basic block from the stack
Variable Renaming (2/3)

• Why preorder traversal
  – If a variable has two definitions in different paths
    • A Phi node would be inserted
    • The two names for the definitions would be propagated from its predecessors
  – If a variable is defined only in the dominators
    • The top of the stack is the name of the latest definition
Variable Renaming: Example

BB0

\[
\begin{align*}
  a &= \text{Phi}(a,a) \\
  b &= \text{Phi}(b,b) \\
  c &= \text{Phi}(c,c) \\
  d &= \text{Phi}(d,d) \\
  i &= \text{Phi}(i,i)
\end{align*}
\]

BB1

\[
\begin{align*}
  a &= \\
  c &= \\
  d &= \text{Phi}(d,d) \\
  i &= \text{Phi}(i,i)
\end{align*}
\]

BB2

\[
\begin{align*}
  b &= \\
  c &= \\
  d &= \text{Phi}(d,d)
\end{align*}
\]

BB3

\[
\begin{align*}
  a &= \\
  d &= \text{Phi}(d,d)
\end{align*}
\]

BB4

\[
\begin{align*}
  d &= \text{Phi}(d,d)
\end{align*}
\]

BB5

\[
\begin{align*}
  c &= \text{Phi}(c,c)
\end{align*}
\]

BB6

\[
\begin{align*}
  b &= \text{Phi}(b,b) \\
  c &= \text{Phi}(c,c) \\
  d &= \text{Phi}(d,d)
\end{align*}
\]

BB7

\[
\begin{align*}
  i &= \text{Phi}(i,i)
\end{align*}
\]

\[
\begin{align*}
  \text{var:} & \quad a \quad b \quad c \quad d \quad i \\
  \text{ctr:} & \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
  \text{stk:} & \quad a0 \quad b0 \quad c0 \quad d0 \quad i0
\end{align*}
\]
Variable Renaming: Example

BB0

\[ a = \Phi(a_0, a) \]
\[ b = \Phi(b_0, b) \]
\[ c = \Phi(c_0, c) \]
\[ d = \Phi(d_0, d) \]
\[ i = \Phi(i_0, i) \]

var: a b c d i
ctr: 1 1 1 1 1
stk: a0 b0 c0 d0 i0

BB1

\[ a = \]
\[ c = \]

BB2

\[ b = \]
\[ c = \]
\[ d = \]

BB3

\[ a = \]
\[ d = \]

BB4

\[ d = \]

BB5

\[ c = \]
\[ c = \Phi(c, c) \]
\[ d = \Phi(d, d) \]

BB6

\[ b = \]
\[ c = \Phi(c, c) \]
\[ d = \Phi(d, d) \]

BB7

\[ i = \]
\[ a = \Phi(a, a) \]
\[ b = \Phi(b, b) \]
\[ c = \Phi(c, c) \]
\[ d = \Phi(d, d) \]
Variable Renaming: Example

BB0

\[
\begin{align*}
    a0 &= \\
    b0 &= \\
    c0 &= \\
    i0 &= \\
\end{align*}
\]

\[
\begin{align*}
    a1 &= \Phi(a0, a) \\
    b1 &= \Phi(b0, b) \\
    c1 &= \Phi(c0, c) \\
    d1 &= \Phi(d0, d) \\
    i1 &= \Phi(i0, i)
\end{align*}
\]

BB1

\[
\begin{align*}
    a2 &= \\
    c2 &= \\
\end{align*}
\]

BB2

\[
\begin{align*}
    b &= \\
    c &= \\
    d &= \\
\end{align*}
\]

BB3

\[
\begin{align*}
    a &= \\
    d &= \\
\end{align*}
\]

BB4

\[
\begin{align*}
    d &= \\
\end{align*}
\]

BB5

\[
\begin{align*}
    c &= \\
\end{align*}
\]

BB6

\[
\begin{align*}
    b &= \\
    c &= \Phi(c, c) \\
    d &= \Phi(d, d)
\end{align*}
\]

BB7

\[
\begin{align*}
    i &= \\
\end{align*}
\]

\[
\begin{align*}
    a &= \Phi(a, a) \\
    b &= \Phi(b, b) \\
    c &= \Phi(c, c) \\
    d &= \Phi(d, d)
\end{align*}
\]

var: a b c d i
ctr: 3 2 3 2 2
stk: a0 b0 c0 d0 i0
     a1 b1 c1 d1 i1
     a2 c2

Thursday, April 18, 2013
Variable Renaming: Example

BB0

\[ a_0 = \]
\[ b_0 = \]
\[ c_0 = \]
\[ i_0 = \]

BB1

\[ a_2 = \]
\[ c_2 = \]
\[ a_1 = \Phi(a_0,a) \]
\[ b_1 = \Phi(b_0,b) \]
\[ c_1 = \Phi(c_0,c) \]
\[ d_1 = \Phi(d_0,d) \]
\[ i_1 = \Phi(i_0,i) \]

BB2

\[ b_2 = \]
\[ c_3 = \]
\[ d_2 = \]

BB3

\[ a = \]
\[ d = \]

BB4

\[ d = \]

BB5

\[ c = \]
\[ c = \Phi(c,c) \]
\[ d = \Phi(d,d) \]

BB6

\[ b = \]

BB7

\[ i = \]
\[ a = \Phi(a_2,a) \]
\[ b = \Phi(b_2,b) \]
\[ c = \Phi(c_3,c) \]
\[ d = \Phi(d_2,d) \]

var: a b c d i
ctr: 3 3 4 3 2
stk: a0 b0 c0 d0 i0
      a1 b1 c1 d1 i1
      a2 b2 c2 d2 c3
Variable Renaming: Example

BB0

BB1

BB2

BB3

BB4

BB5

BB6

BB7

Var: a b c d i

Ctr: 3 3 4 3 2

Stk: a0 b0 c0 d0 i0

a1 = Phi(a0,a)
b1 = Phi(b0,b)
c1 = Phi(c0,c)
d1 = Phi(d0,d)
i1 = Phi(i0,i)

This just updates the stack to remove the stuff from the left path out of BB1.

Pop names after BB2
Variable Renaming: Example

BB0

a0 =
b0 =
c0 =
i0 =

a1 = Phi(a0,a)
b1 = Phi(b0,b)
c1 = Phi(c0,c)
d1 = Phi(d0,d)
i1 = Phi(i0,i)

BB1

a2 =
c2 =

BB2

b2 =
c3 =
d2 =

BB3

a3 =
d3 =

BB4

d =

BB5

c =

BB6

b =

BB7

i =

var: a b c d i
ctr: 4 3 4 4 2
stk: a0 b0 c0 d0 i0
a1 b1 c1 d1 i1
a2 c2 d3

a = Phi(a2,a)
b = Phi(b2,b)
c = Phi(c3,c)
d = Phi(d2,d)
Variable Renaming: Example

\begin{itemize}
    \item \textbf{BB0:}
        \begin{align*}
            a_0 &= \Phi(a_0, a) \\
            b_0 &= \Phi(b_0, b) \\
            c_0 &= \Phi(c_0, c) \\
            i_0 &= \Phi(i_0, i)
        \end{align*}

    \item \textbf{BB1:}
        \begin{align*}
            a_2 &= \Phi(a_0, a) \\
            c_2 &= \Phi(b_0, b) \\
            b_2 &= \Phi(c_0, c) \\
            d_1 &= \Phi(c_0, c) \\
            i_1 &= \Phi(i_0, i)
        \end{align*}

    \item \textbf{BB2:}
        \begin{align*}
            b_2 &= \Phi(a_2, a) \\
            c_3 &= \Phi(b_0, b) \\
            d_2 &= \Phi(c_0, c)
        \end{align*}

    \item \textbf{BB3:}
        \begin{align*}
            a_3 &= \Phi(a_2, a) \\
            d_3 &= \Phi(b_0, b) \\
            c &= \Phi(c_0, c)
        \end{align*}

    \item \textbf{BB4:}
        \begin{align*}
            d_4 &= \Phi(a_0, a) \\
            b &= \Phi(b_0, b)
        \end{align*}

    \item \textbf{BB5:}
        \begin{align*}
            c &= \Phi(c_2, c) \\
            d &= \Phi(d_4, d)
        \end{align*}

    \item \textbf{BB6:}
        \begin{align*}
            b &= \Phi(a_2, a) \\
            d &= \Phi(b_2, b) \\
            c &= \Phi(c_3, c)
        \end{align*}

    \item \textbf{BB7:}
        \begin{align*}
            i &= \Phi(a_0, a) \\
            b &= \Phi(b_0, b)
        \end{align*}
\end{itemize}

\begin{itemize}
    \item \textbf{Variables:}
        \begin{align*}
            \text{var: } a & \quad b & \quad c & \quad d & \quad i \\
            \text{ctr: } 4 & \quad 3 & \quad 4 & \quad 5 & \quad 2 \\
            \text{stk: } a_0 & \quad b_0 & \quad c_0 & \quad d_0 & \quad i_0
        \end{align*}

        \begin{align*}
            a_1 &= \Phi(a_0, a) \\
            b_1 &= \Phi(b_0, b) \\
            c_1 &= \Phi(c_0, c) \\
            d_1 &= \Phi(d_0, d) \\
            i_1 &= \Phi(i_0, i)
        \end{align*}
\end{itemize}
Variable Renaming: Example

- BB0
  - $a_0 = \Phi(a_0, a)$
  - $b_0 = \Phi(b_0, b)$
  - $c_0 = \Phi(c_0, c)$
  - $i_0 = \Phi(i_0, i)$

- BB1
  - $a_2 = \Phi(a_2, a)$
  - $c_2 = \Phi(c_2, c)$
  - $b_1 = \Phi(b_0, b)$
  - $d_1 = \Phi(d_0, d)$
  - $i_1 = \Phi(i_0, i)$

- BB2
  - $b_2 = \Phi(b_2, b)$
  - $c_3 = \Phi(c_3, c)$
  - $d_2 = \Phi(d_2, d)$

- BB3
  - $a_3 = \Phi(a_3, a)$
  - $d_3 = \Phi(d_3, d)$
  - $c_4 = \Phi(c_4, c)$

- BB4
  - $d_4 = \Phi(d_4, d)$

- BB5
  - $c_4 = \Phi(c_2, c_4)$
  - $d = \Phi(d_4, d_3)$

- BB6
  - $b = \Phi(b_2, b)$

- BB7
  - $i = \Phi(i_0, i)$

- var: $a$, $b$, $c$, $d$, $i$
- ctr: 4, 3, 5, 5, 2
- stk: $a_0$, $b_0$, $c_0$, $d_0$, $i_0$
Variable Renaming: Example

BB0

\begin{align*}
a0 &= \\
b0 &= \\
c0 &= \\
i0 &= \\
a1 &= \Phi(a0,a) \\
b1 &= \Phi(b0,b) \\
c1 &= \Phi(c0,c) \\
d1 &= \Phi(d0,d) \\
i1 &= \Phi(i0,i)
\end{align*}

BB1

\begin{align*}
a2 &= \\
c2 &= \\
\end{align*}

BB2

\begin{align*}
b2 &= \\
c3 &= \\
d2 &= \\
\end{align*}

BB3

\begin{align*}
a3 &= \\
d3 &= \\
\end{align*}

BB4

\begin{align*}
d4 &= \\
\end{align*}

BB5

\begin{align*}
c4 &= \\
\end{align*}

BB6

\begin{align*}
b3 &= \\
c5 &= \Phi(c2,c4) \\
d5 &= \Phi(d4,d3)
\end{align*}

BB7

\begin{align*}
i &= \\
a &= \Phi(a2,a3) \\
b &= \Phi(b2,b3) \\
c &= \Phi(c3,c5) \\
d &= \Phi(d2,d5)
\end{align*}

var: a b c d i
ctr:  4  4  6  6  2
stk: a0 b0 c0 d0 i0
      a1 b1 c1 d1 i1
      a2 b3 c2 d3
      a3 c5 d5

Thursday, April 18, 2013
Variable Renaming: Example

\[ a_2 = c_2 = b_2 = c_3 = d_2 = a_3 = d_3 = c_4 = d_4 = b_3 = i_2 = \]

\[ a_1 = \text{Phi}(a_0, a_4) \]
\[ b_1 = \text{Phi}(b_0, b_4) \]
\[ c_1 = \text{Phi}(c_0, c_6) \]
\[ d_1 = \text{Phi}(d_0, d_6) \]
\[ i_1 = \text{Phi}(i_0, i_2) \]
\[ c_5 = \text{Phi}(c_2, c_4) \]
\[ d_5 = \text{Phi}(d_4, d_3) \]

\[ a_4 = \text{Phi}(a_2, a_3) \]
\[ b_4 = \text{Phi}(b_2, b_3) \]
\[ c_6 = \text{Phi}(c_3, c_5) \]
\[ d_6 = \text{Phi}(d_2, d_5) \]

var: \ a \ b \ c \ d \ i \\
ctr: \ 5 \ 5 \ 7 \ 7 \ 3 \\
stk: \ a_0 \ b_0 \ c_0 \ d_0 \ i_0 \\\
\ a_1 \ b_1 \ c_1 \ d_1 \ i_1 \ a_2 \ b_4 \ c_2 \ d_6 \ i_2 \ a_4 \ c_6 \]
Thanks & all the best!