Fractals, mountains, and trees

- Coastlines
- Snowflakes
- Sponges
- Mountains, terrains
- Trees, bushes
Real Coastline
Real Coastline

9/10/98
Real Coastline

© 2000 GlobeXplorer, VantagePoint Network
Real Coastline

- Zooming in
How to represent and store all that?
- Cubic Bezier curves?
- Or do we generate it on the fly maybe?
- Koch curve:
  
  What is the length of this curve?
  
  $1, 4*(1/3), 16*(1/9), 64*(1/27), \ldots \left(\frac{4}{3}\right)^n \ldots \infty$
• Let’s go back to simple shapes…
  - line: dimension $d = 1$
    - $1, 2*(1/2)^1, 4*(1/4)^1, \ldots$
  - square: dimension $d = 2$
    - $1, 4*(1/2)^2, 16*(1/4)^2, \ldots$

Fractal dimension

- Line: dimension $d = 1$
  - scaling factor $s = 1/2$
  - number of subparts $n = 2$
- Rectangle: dimension $d = 2$
  - $s = 1/2$
  - $n = 4$
- In general:
  - $ns^d = 1$
  - $d = \log n / \log(1/s)$
Fractal dimension

Koch curve:
\[ n = 4, \ s = 1/3, \ \text{so} \ d = \log 4 / \log 3 \approx 1.26 \]

L-systems

• How can we produce such objects?
• L-system is
  - symbols
    - language describing a 2d/3d scene
  - an axiom
    - starting point
  - rewriting rules
L-systems

- Lindenmayer 1968
- Turtle graphics (Seymour Papert)
  - F draw forward
  - f move forward
  - + turn left
  - - turn right
  - [ push current state onto stack
  - ] pop current state from the stack

Koch l-system

- F+F--F+F
  \[ \text{angle}=\frac{2\pi}{6} \]

Koch 
  \{
  \text{Angle 6}
  \text{Axiom F}
  \text{F=F+F--F+F}
  \}
Generating Koch’s snowflake

- Start: F
- Generation 1:
  - F+F--F+F

- Generation 2:
  - F+F--F+F+F+F--F+F--F+F+F+F+F--F+F--F+F+F+F+F--F+F

Example in 2d

Example {
  Angle 16
  Axiom ++++FS
  S=+[FS]-[FS]-[FS]
}
Koch Island

KochIsland {
  Angle 4
  Axiom F+F+F+F
  F=F+F-F-F+F+F-F
}

Plants in 3d

- Similarly:
  - 3d transforms
  - rotations
  - nested transforms
  - colors
  - position

- Przemyslaw Prusinkiewicz, U. of Calgary
Terrain modeling

- Fractal mountains
  - geometry
  - colors
  - vegetation
Fractional Brownian Motion

Add random values at finer and finer scales
Fractional Brownian Motion

- Additional parameters…

2D: Creating fractal mountains

- Start with planar triangulation
- Let user displace coarse triangles
- Recursively subdivide and displace randomly
Elevation

- Elevation governs:
  - color
    - snow
    - grass
  - trees distribution
  - roughness

Sky, clouds

- Cloud texture

\[ I(x, y) = \sum_{i=1}^{n} c_i \sin(\omega^x_i x + p^x_i) \sum_{i=1}^{n} c_i \sin(\omega^y_i y + p^y_i) \]

\[ \omega^x_{i+1} = 2\omega^x_i \]
\[ \omega^y_{i+1} = 2\omega^y_i \]
\[ c_{i+1} = 0.707c_i \]