Lecture 16

## Fractals, mountains, and trees

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## Fractals

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- Coastlines
- Snowflakes
- Sponges
- Mountains, terrains
- Trees, bushes

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## Real Coastline

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## Real Coastline

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## Real Coastline

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## Real Coastline

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## Real Coastline

Lecture 16


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## Real Coastline

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## Real Coastline

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## Real Coastline

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## Real Coastline

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## Real Coastline

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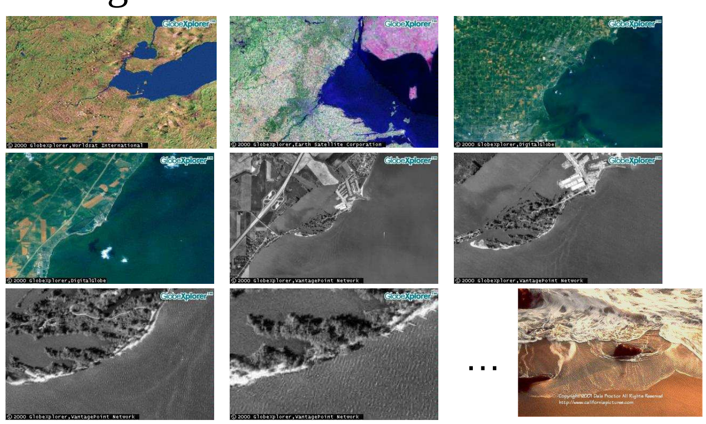
- Zooming in



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## (Fake) Fractal Coastline

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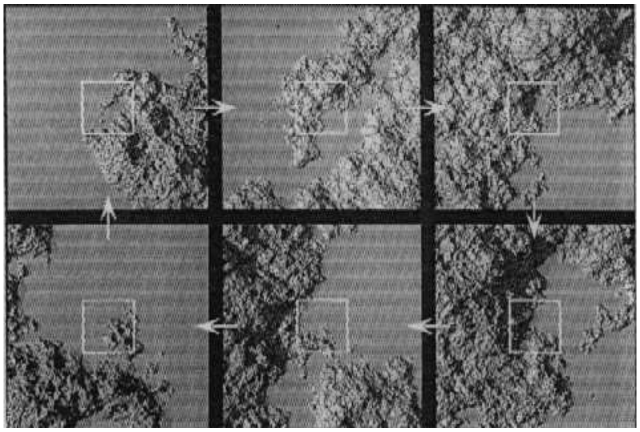

Voss, R.F., (1989), Random fractals: Self-affinity in noise, music, mountains and clouds
in Ammon Aharony and Jens Feder (eds),
Physica D (Non-linear Phenomena): Fractals in Physics, Vol. 88, pp. 362-371.

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## Coastline

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- How to represent and store all that?
  - Cubic Bezier curves?
- Or do we generate it on the fly maybe?
- Koch curve:

  - What is the length of this curve?
  - 1, 4*(1/3), 16*(1/9), 64*(1/27), … $\left(\frac{4}{3}\right)^n$… $\infty$

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## Fractal dimension

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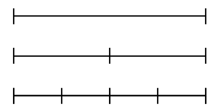
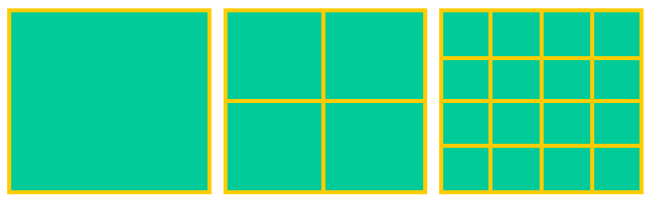
- Let's go back to simple shapes…
  - line: dimension = 1
    - 1, 2*(1/2)$^1$,4*(1/4)$^1$, …
  - square: dimension = 2
    - 1, 4*(1/2)$^2$, 16*(1/4)$^2$, …





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## Fractal dimension

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- Line: dimension d = 1

  scaling factor s = 1/2

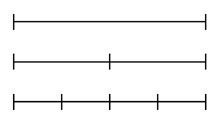
  number of subparts n = 2
- Rectangle: dimension d = 2

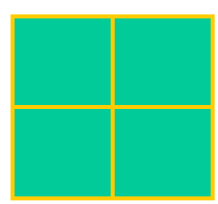
  s = 1/2

  n = 4
- In general:

  $ns^d = 1$

  d = log n / log(1/s)





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## Fractal dimension

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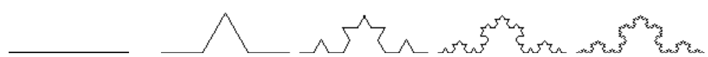
Koch curve:

n = 4, s = 1/3, so d = log 4 / log 3 ≈ 1.26



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## L-systems

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- How can we produce such objects?
- L-system is
  - symbols
    - language describing a 2d/3d scene
  - an axiom
    - starting point
  - rewriting rules

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## L-systems

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- Lindenmayer 1968
- Turtle graphics (Seymour Papert)
  - F draw forward
  - f move forward
  - + turn left
  - - turn right
  - [ push current state onto stack
  - ] pop current state from the stack

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## Koch l-system

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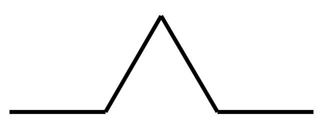
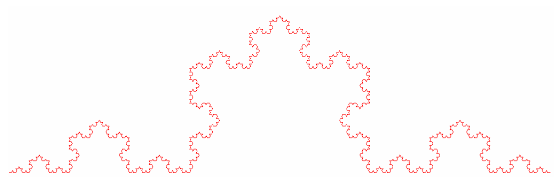
- F+F--F+F
  angle=(2π)/6



```
Koch {
  Angle 6
  Axiom F
  F=F+F--F+F
}
```



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## Generating Koch's snowflake

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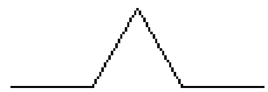
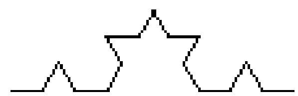
- Start: F
- Generation 1:
  - F+F--F+F



Koch {
  Angle 6
  Axiom F
  F=F+F--F+F
}

- Generation 2:
  - F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F



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## Example in 2d

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Example {
  Angle 16
  Axiom ++++FS
  S=+[FS]-[FS]-[FS]
}

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## Koch Island

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```
KochIsland {
  Angle 4
  Axiom F+F+F+F
  F=F+F-F-FF+F+F-F
}
```

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## Plants in 3d

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- Similarly:
  - 3d transforms
  - rotations
  - nested transforms
  - colors
  - position

- Przemyslaw Prusinkiewicz, U. of Calgary

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**Terrain modeling**

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- Fractal mountains
  - geometry
  - colors
  - vegetation

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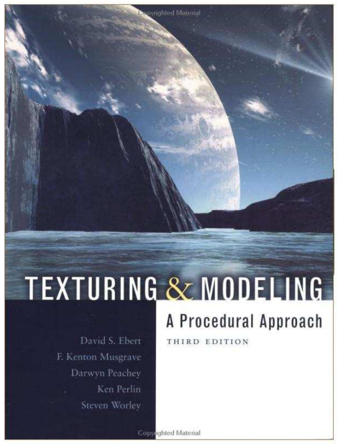


Texturing & Modeling: A Procedural Approach, by Ebert *et al.*

The Science of Fractal Images
by Heinz-Otto Peitgen, Dietmar Saupe (Eds)

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## Fractional Brownian Motion

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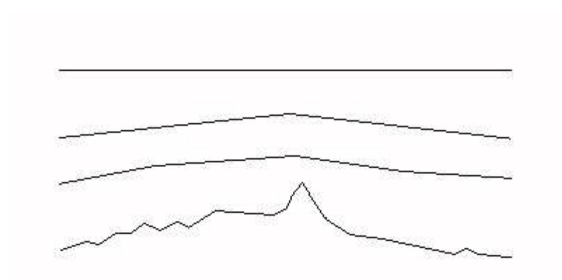
Add random values at finer and finer scales



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## Fractional Brownian Motion

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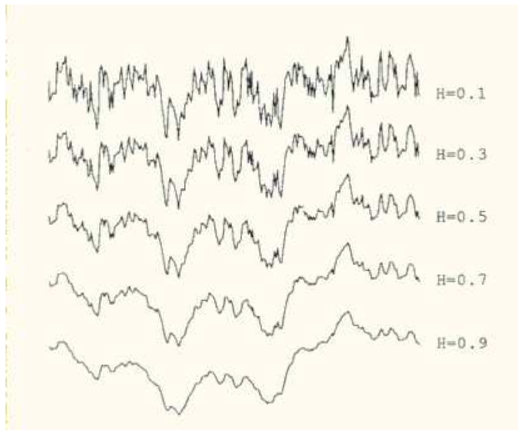
- Additional parameters…

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## 2D: Creating fractal mountains

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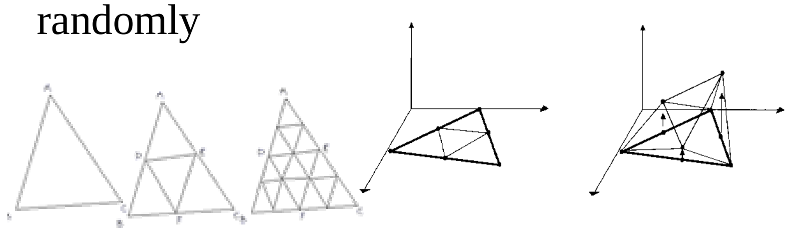
- Start with planar triangulation
- Let user displace coarse triangles
- Recursively subdivide and displace randomly



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## Elevation

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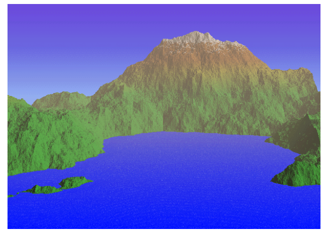
- Elevation governs:
  - color
    - snow
    - grass
  - trees distribution
  - roughness





## Sky, clouds

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- Cloud texture

$$I(x,y) = \sum_{i=1}^{n} c_i \sin(\omega_i^x x + p_i^x) \sum_{i=1}^{n} c_i \sin(\omega_i^y y + p_i^y)$$

$$\omega_{i+1}^x = 2\omega_i^x$$

$$\omega_{i+1}^y = 2\omega_i^y$$

$$c_{i+1} = 0.707 c_i$$