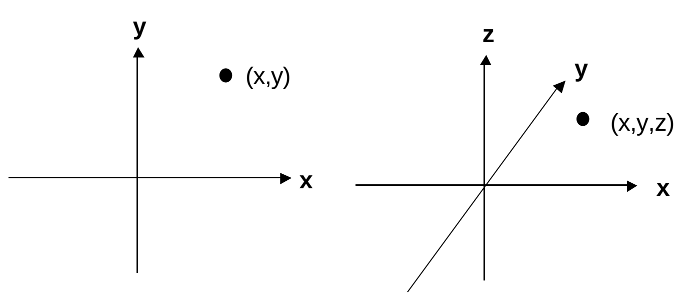
Interactive Computer Graphics

Lecture 2

- Coordinate Systems
- Vectors and Points
- Matrices
- Clipping

Coordinate Systems

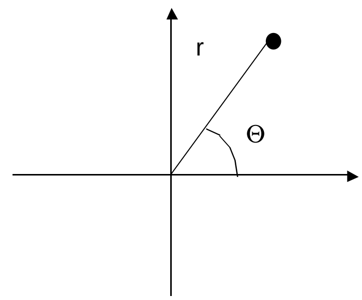
Lecture 2



2D Cartesian 3D Cartesian

Coordinate Systems

Lecture 2



Polar

$$x = r\cos\Theta$$
$$y = r\sin\Theta$$

$$r = \sqrt{x^2 + y^2}$$
$$\Theta = \tan^{-1}\left(\frac{y}{x}\right)$$

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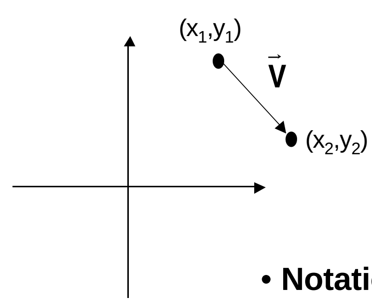
Points

Lecture 2

Point (x,y) : Absolute Location



Vector form two Points

$$\vec{\mathbf{V}} = \langle (x_2 - x_1), (y_2 - y_1) \rangle$$

- **Notation**

Points ( )
Vectors < >
Matrices [ ]

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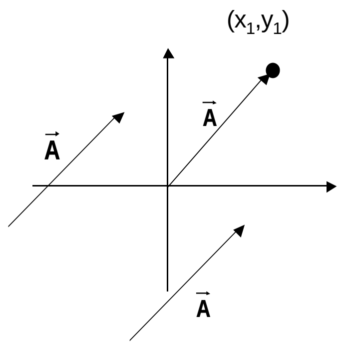
Vectors

Lecture 2

Vector <x,y> : Relative Direction



$$\vec{\mathbf{A}} = A_x \hat{\mathbf{x}} + A_y \hat{\mathbf{y}}$$

$$\vec{\mathbf{A}} = \langle A_{x1}, A_{y1} \rangle$$

Magnitude of Vector $\vec{\mathbf{V}}$: $\|\vec{\mathbf{V}}\| = \sqrt{V_x^2 + V_y^2}$

Unit Vector : $\hat{\mathbf{v}} = \dfrac{\vec{\mathbf{V}}}{\|\vec{\mathbf{V}}\|}$

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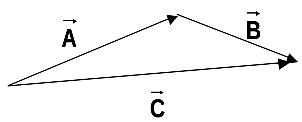
Vector Addition

Lecture 2

•Tip-to-Tail



$$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = \langle A_x + B_x, A_y + B_y \rangle$$

• Vector Properties

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$$

$$(-1)\vec{\mathbf{A}} = -\vec{\mathbf{A}} = \langle -A_x, -A_y \rangle$$

$$(\vec{\mathbf{A}} + \vec{\mathbf{B}}) + \vec{\mathbf{C}} = \vec{\mathbf{A}} + (\vec{\mathbf{B}} + \vec{\mathbf{C}})$$

$$(s + t)\vec{\mathbf{A}} = s\vec{\mathbf{A}} + t\vec{\mathbf{A}}$$

$$s\vec{\mathbf{A}} = \langle sA_x, sA_y \rangle$$

$$s(\vec{\mathbf{A}} + \vec{\mathbf{B}}) = s\vec{\mathbf{A}} + s\vec{\mathbf{B}}$$

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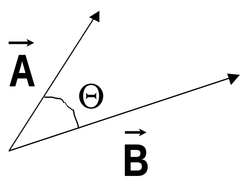
Vector Dot Product

Lecture 2

$$\text{Vector} \cdot \text{Vector} = \text{Scalar}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \quad = \left\|\vec{A}\right\| \left\|\vec{B}\right\| \cos\Theta \qquad 0 \le \Theta \le p$$



$$\Theta = \cos^{-1}\left(\frac{\vec{A} \cdot \vec{B}}{\left\|\vec{A}\right\| \left\|\vec{B}\right\|}\right)$$

Important case :

$$\vec{A} \cdot \vec{B} = 0 \quad \Rightarrow \quad \cos\Theta = 0 \quad \Rightarrow \quad \Theta = 90^\circ \qquad \therefore \; \vec{A} \perp \vec{B}$$

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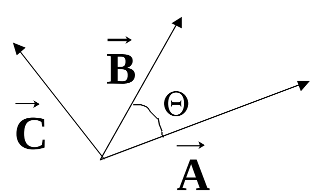
Vector Cross Product

Lecture 2

$$\text{Vector} \times \text{Vector} = \text{Vector}$$

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \left\langle (A_y B_z - A_z B_y), (A_z B_x - A_x B_z), (A_x B_y - A_y B_x) \right\rangle = \vec{\mathbf{C}}$$



$$\Rightarrow \left\|\vec{\mathbf{A}}\right\| \left\|\vec{\mathbf{B}}\right\| \sin\Theta \qquad 0 \le \Theta \le p$$

Note:
C is perpendicular to the plane made by A and B

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y)\hat{\mathbf{x}} - (A_x B_z - A_z B_x)\hat{\mathbf{y}} + (A_x B_y - A_y B_x)\hat{\mathbf{z}}$$

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Vector Cross Product

Lecture 2

•Properties

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} \neq \vec{\mathbf{B}} \times \vec{\mathbf{A}} \qquad \vec{\mathbf{A}} \times \vec{\mathbf{B}} = -(\vec{\mathbf{B}} \times \vec{\mathbf{A}})$$

$$\left(\vec{\mathbf{A}} \times \vec{\mathbf{B}}\right) \times \vec{\mathbf{C}} \neq \vec{\mathbf{A}} \times \left(\vec{\mathbf{B}} \times \vec{\mathbf{C}}\right)$$

$$\vec{\mathbf{A}} \times \left(\vec{\mathbf{B}} + \vec{\mathbf{C}}\right) = \vec{\mathbf{A}} \times \vec{\mathbf{B}} + \vec{\mathbf{A}} \times \vec{\mathbf{C}}$$

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Matrices

Lecture 2

• A Rectangular Array of Numbers

$$[\mathbf{A}] = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ A_{m1} & A_{m2} & \dots & A_{mn} \end{bmatrix}$$

Columns

Rows

• Matrix Size Given as (rows x columns) = *m* x *n*

• Individual Elements Referenced by : $A_{\text{row col}}$

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Matrix Operations

Lecture 2

Scalar Multiplication :

$$s[\mathbf{A}] = \begin{bmatrix} sA_{11} & sA_{12} & \dots & sA_{1n} \\ sA_{21} & sA_{22} & \dots & sA_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ sA_{m1} & sA_{m2} & \dots & sA_{mn} \end{bmatrix}$$

Matrix Addition : $[\mathbf{A}] + [\mathbf{B}] = \begin{bmatrix} A_{11}+B_{11} & A_{12}+B_{12} & \dots & A_{1n}+B_{1n} \\ A_{21}+B_{21} & A_{22}+B_{22} & \dots & A_{2n}+B_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ A_{m1}+B_{21} & A_{m2}+B_{m2} & \dots & A_{mn}+B_{mn} \end{bmatrix}$

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Matrix Operations

Lecture 2

Matrix Multiplication : $[\mathbf{C}] = [\mathbf{A}][\mathbf{B}]$ $\quad$ *where* $\quad C_{ij} = \sum_{k=1}^{n} (A_{ik})(B_{kj})$

Note: number of Columns in A must match number of Rows in B

$$[C] = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ A_{m1} & A_{m2} & \dots & A_{mn} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1n} \\ B_{21} & B_{22} & \dots & B_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ B_{m1} & B_{m2} & \dots & B_{mn} \end{bmatrix}$$

$$C_{11} = \sum_{k=1}^{n} (A_{1k})(B_{k1}) = (A_{11})(B_{11}) + (A_{12})(B_{21}) + (A_{13})(B_{31})\dots + \dots(A_{1n})(B_{m1})$$

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Matrix Operations

Lecture 2

Determinant : $\|[\mathbf{A}]\| = \det[\mathbf{A}]$ If [A] is a 2x2 matrix $\|[\mathbf{A}]\| = A_{11}A_{22} - A_{12}A_{21}$

Matrix Transpose : $[\mathbf{A}]^T \Rightarrow$ Rows become columns

$$[\mathbf{A}] = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ A_{m1} & A_{m2} & \dots & A_{mn} \end{bmatrix} \qquad [\mathbf{A}]^T = \begin{bmatrix} A_{11} & A_{21} & \dots & A_{m1} \\ A_{12} & A_{22} & \dots & A_{m2} \\ \vdots & \vdots & \vdots & \vdots \\ A_{1n} & A_{2n} & \dots & A_{mn} \end{bmatrix}$$

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Matrices & Vectors

Lecture 2

- Matrix Form of a Vector

Row Vector : $\vec{\mathbf{A}} = [\mathbf{A}] = [A_x \quad A_y \quad A_z]$ Column Vector : $\vec{\mathbf{A}} = [\mathbf{A}] = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$

Matrix Form of Dot Product :

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = [\mathbf{A}]^T [\mathbf{B}] = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}^T \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = [A_x \quad A_y \quad A_z] \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = A_x B_x + B_y B_y + A_z B_z$$

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Clipping

Lecture 2

Why do we need clipping?

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EECS 487

Clipping

Lecture 2

Why do we need clipping?

- So we can define objects based on points inside our display window.
- To extract portions of an object database.
- Reduce computations by only processing displayable objects
- To prevent unwanted mathematical wrapping.

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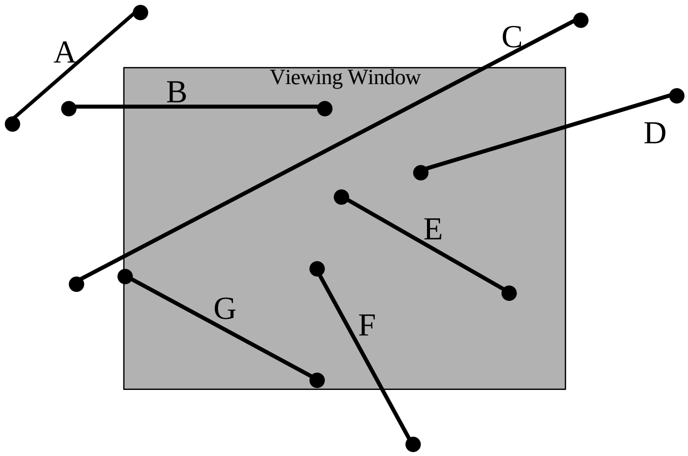
Clipping

Lecture 2



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Clipping

Lecture 2

Where in the graphics pipeline should we clip?

- During the display list building process.
- Before the objects are rendered.

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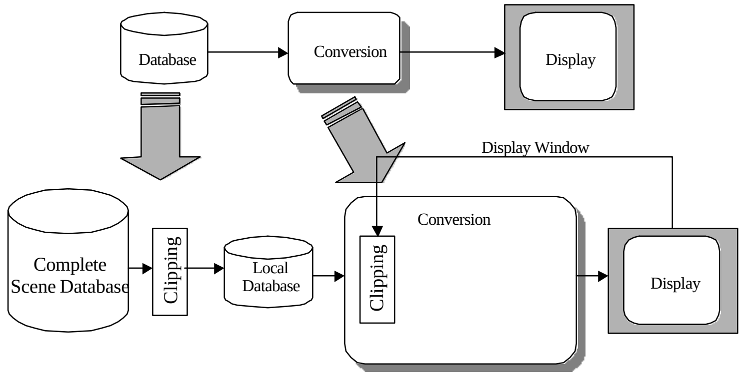
Clipping

Lecture 2

Graphics Pipeline



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Lecture 2

Simultaneous Equations

- Sutherland - Cohen
- Cyrus - Beck

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Line Clipping Simultaneous Equations

Lecture 2

For each line in the database

- Find the intersection of two mathematically infinite lines. The line under test and a line representing each edge of the viewing window.
- Test each intersection to see if it is interior to the viewing window.

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Line Clipping Simultaneous Equations

Lecture 2

We don't want to use the simple analytic geometry definition of a line $y = a * x + b$

We will use a parametric formulation:

$$
\begin{aligned}
x &= x_0 + t(x_1 - x_0) \\
y &= y_0 + t(y_1 - y_0)
\end{aligned}
\qquad (0 \le t \le 1)
$$

We then solve two sets of simultaneous equations

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Line Clipping Simultaneous Equations

Lecture 2

Problems:

- A parallel line test must be performed before solving the simultaneous equations

- Requires considerable calculations

  - It is very inefficient

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Line Clipping Sutherland - Cohen

Lecture 2

•Test endpoint pairs of each line for trivial acceptance or rejection.

•Lines that did not meet the above test are divided into two lines at the window edge.

•Step one and two are repeated until all the lines have been accepted or rejected.

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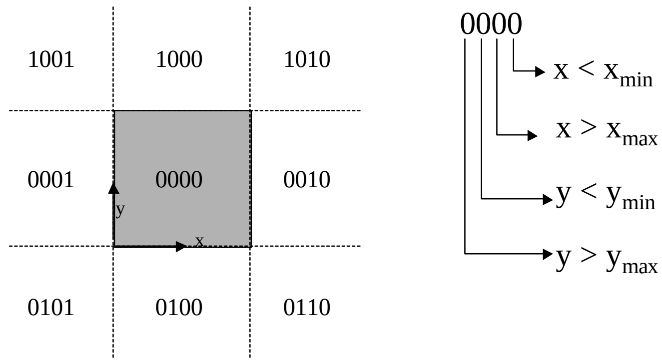
Line Clipping Sutherland - Cohen

Lecture 2

Trivial Accept/Reject test:



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Line Clipping Sutherland - Cohen

Lecture 2

Trivial Accept/Reject test (continued):

Accept if $P_0 + P_1 = 0$

Reject if P_0 & P_1 != 0

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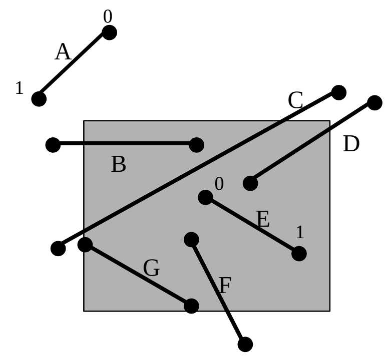
Line Clipping Sutherland - Cohen

Lecture 2



$E_0 = 0000$

$E_1 = 0000$

$E_0 + E_1 = 0$ E is Accepted

$A_0 = 1000$

$A_1 = 1001$

$A_0 + A_1 = 0001$

$A_0 \& A_1 = 1000$ A is Rejected

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Line Clipping Sutherland - Cohen

Lecture 2

Lines that are left must be subdivided.

Use the bit code to determine which edge is to be used to sub-divide the line.

Use the line equation to find the edge intersection point.

For Left and Right $y = a*x_{L \text{ or } R} + b$

For Top and Bottom $x = a*y_{T \text{ or } B} + b$

Order in which edge test are done is important.

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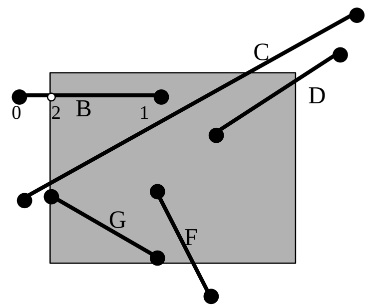
Line Clipping Sutherland - Cohen

Lecture 2



B_0 = 0001

B_1 = 0000

B_2 = 0000

B2 + B1 = 0000 Accept

B0 to B2 is automatically Rejected

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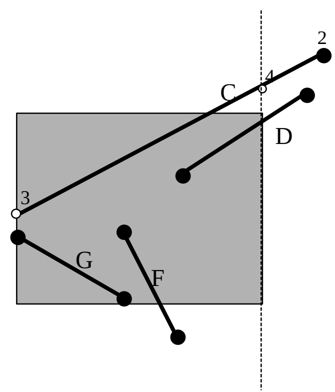
Line Clipping Sutherland - Cohen

Lecture 2



C_2 = 1010

C_4 = 1000

C_3 = 0000

C4 + C3 = 1000

C4 & C3 = 0000 Still no solution!

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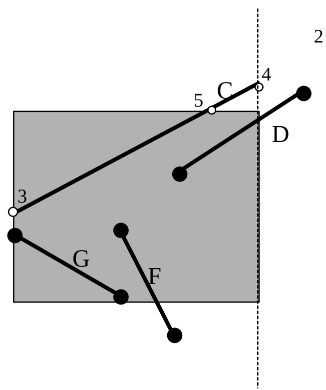
Line Clipping Sutherland - Cohen

Lecture 2



C_4 = 1000

C_5 = 0000

C_3 = 0000

C5 + C3 = 0000 Accepted

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Line Clipping Cyrus - Beck

Lecture 2

Start with the Trivial Accept/Reject test

Use the parametric line equation and the normal of the testing edge to find true edge intersection.

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Line Clipping Cyrus - Beck

Lecture 2

$\vec{N}_i$ is the Normal of the edge being tested

$\vec{D}$ is the vector from P0 to P1 = P1 - P0

P_{Ei} is any point on the edge being tested

If $\vec{N}_i \bullet \vec{D} \neq 0$ the lines are not parallel.

Solve for t: $t = \dfrac{\vec{N}_i \bullet [P_0 - P_{Ei}]}{-\vec{N}_i \bullet \vec{D}}$ for each edge

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Line Clipping Cyrus - Beck

Lecture 2

- For each edge i check t_i

  If $t_i < 0$ or $t_i > 1$ this edge intersection is not on the clipping window.

  Now Classify each point as PE or PL

  $N_i \cdot D < 0$ it is PE

  $N_i \cdot D > 0$ it is PL
- If $t_{smallest}$ for PL is $< t_{largest}$ for PE the line is outside the clipping window
- Otherwise $t_{smallest}$ for PL and $t_{largest}$ for PE are the clipped line bounding points.

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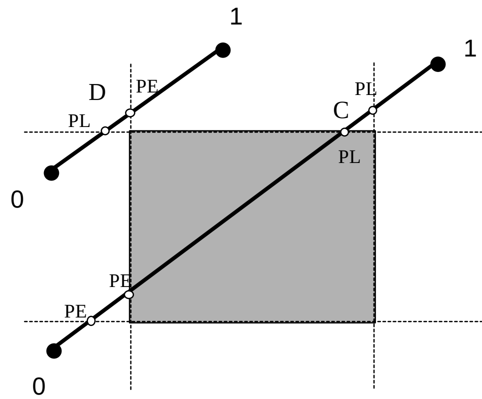
Line Clipping Cyrus - Beck

Lecture 2



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