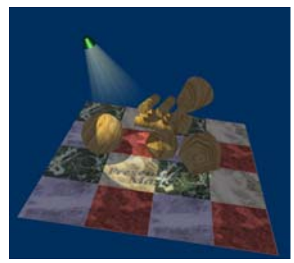
Transforms II

Lecture 4

Overview

- Homogeneous Coordinates
- 3-D Transforms
- Viewing Projections



Homogeneous Coordinates

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$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \\ \frac{z}{w} \end{bmatrix} \qquad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \quad \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \\ \frac{z}{w} \\ 1 \end{bmatrix} \quad \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Allows translations to be included into matrix transform.

Allows us to distinguish between a vector and a point.

In perspective transformations the extra coordinate can be thought to contain the perspective information or scaling.

2

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Homogeneous Coordinates

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$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \\ w \end{bmatrix} \longrightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- As w gets smaller Real space X gets Larger
- When w reaches 0 X is now at infinity
- Homogeneous coordinates allows us to deal mathematically with infinity

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Homogeneous Coordinates

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$$\begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \\ \frac{z}{w} \\ 1 \end{bmatrix} \qquad \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$$

Point

Vector

Many different homogeneous space vectors for the same real space point

Vectors can not be translated but can be scaled and rotated

Points can be translated and blended!

Homogeneous Coordinates

Lecture 4

Attribute	Vector	Point
Represents	Magnitude and Direction	Location
Origin	Unique	Arbitrary
Transformation	Liner Scale and Rotate	Affine Move

$$\begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix} \qquad \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Vector Point

5

3-D Transforms

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Rotation

$$\begin{bmatrix} x''' \\ y''' \\ z''' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ z'' \\ 1 \end{bmatrix}$$

Rotation about X

$$\begin{bmatrix} x'' \\ y'' \\ z'' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & \sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$

Rotation about Y

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\lambda & -\sin\lambda & 0 & 0 \\ \sin\lambda & \cos\lambda & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotation about Z

6

3-D Transforms

Lecture 4

Rotation

$$\begin{bmatrix} x''' \\ y''' \\ z''' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & \sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\lambda & -\sin\lambda & 0 & 0 \\ \sin\lambda & \cos\lambda & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3-D Transforms

Lecture 4

Scale

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} scalex & 0 & 0 & 0 \\ 0 & scaley & 0 & 0 \\ 0 & 0 & scalez & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3-D Transforms

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Translation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

9

3-D Transforms

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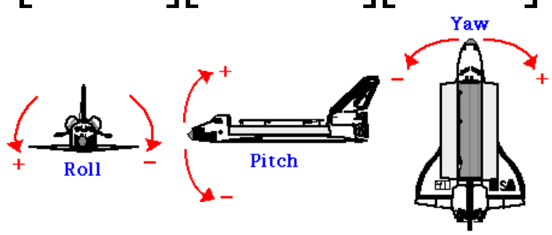
Composite Rotation Matrix

$$[R] = [Roll][Pitch][Yaw]$$



$$[M] = [R][S][T]$$

10

Transform composition

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- Ship moves
- Character moves
- All represented as a sequence of matrix transforms (dependent on time)

11

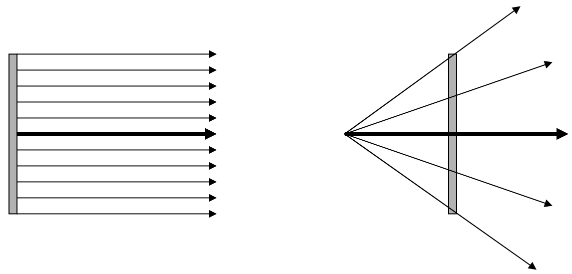
Viewing Projections

Lecture 4

How do we see in 3-D

Parallel Projections

Perspective Projections



12

Viewing Projections

Lecture
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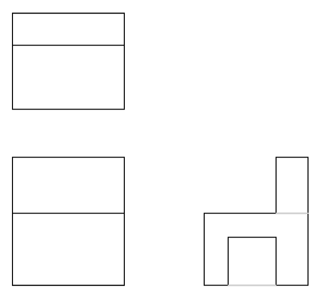
Parallel Projections

Elevations:

Projection plane is perpendicular to a principle axis. Front, Top (Plan), Side.



13

Viewing Projections

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Parallel Projections

Axonometric:

Projection plane is not orthogonal to a principle axis.

Isometric:

Direction of projection makes equal angles with each principal axis.

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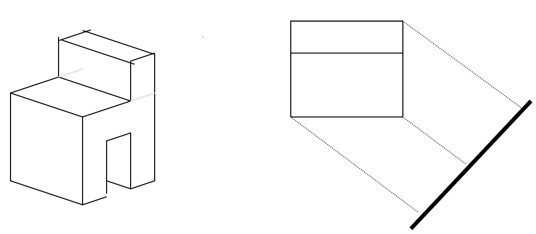

14

Viewing Projections

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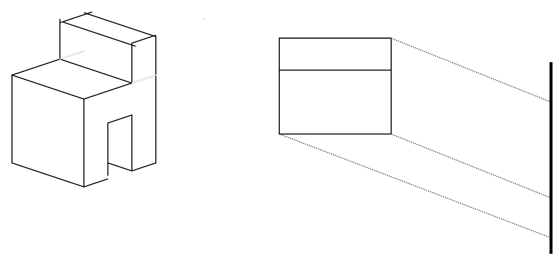
Parallel Projections

Oblique:
Direction of projection is not orthogonal to the projection plane.



15

Viewing Projections

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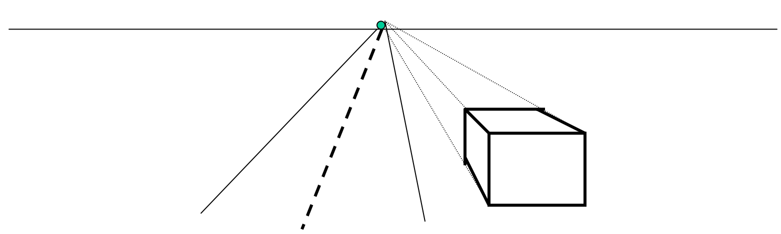
Perspective Projection

One-point:
- One principle axis cut by projection plane.
- One axis vanishing point.



16

Viewing Projections

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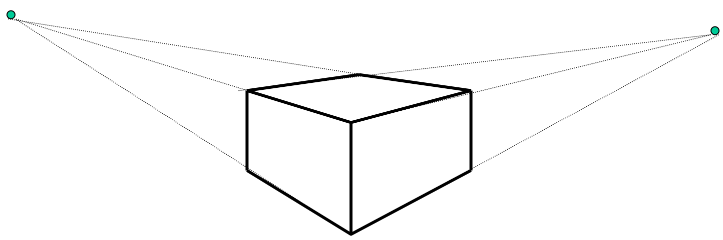
Perspective Projection

Two-point:

- Two principle axes cut by projection plane.
- Two vanishing points.



17

Viewing Projections

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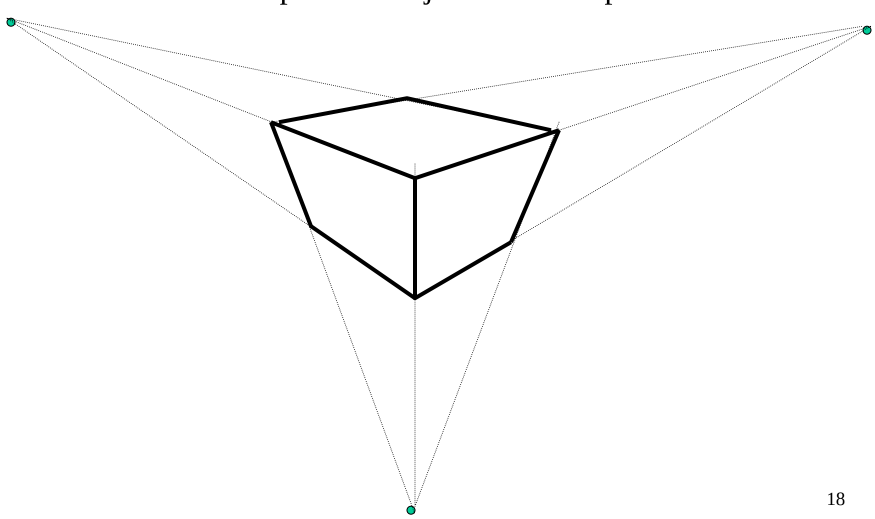
Perspective Projection Three-point



18

Viewing Projections

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Definitions

VRC	-	Viewing Reference Coordinate
VRP	-	View Reference Point
VPN	-	View Plane Normal
VUP	-	View Up Direction
DOP	-	Direction of Projection
PRP	-	Projection Reference Point
		Center of Projection
VP	-	Viewing Plane
BCP	-	Back Clipping Plane
FCP	-	Front Clipping Plane

19

Notation

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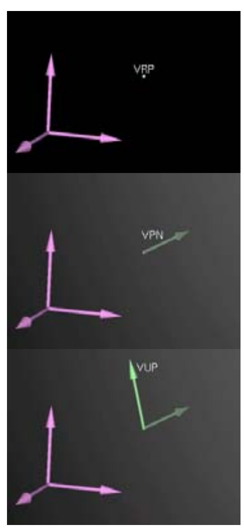
- VRP – camera position
- VPN – view plane normal
- VUP – view up direction



http://www.scs.leeds.ac.uk/cuddles/hyperbks/Rendering/Pipeline/view_def_vref.html

20

Parallel Projections

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1. Translate VRP to the origin

2. Rotate VRC such that n axis (VPN) becomes z, u axis becomes x, and v axis becomes y.

3. Shear such that the direction of projection becomes parallel to the z axis.

4. Translate and scale into the parallel-projection canonical view volume.

21

Parallel Projections

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Step1:

Simply the negative of the VRP vector

$$\begin{bmatrix} 1 & 0 & 0 & -vrp_x \\ 0 & 1 & 0 & -vrp_y \\ 0 & 0 & 1 & -vrp_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

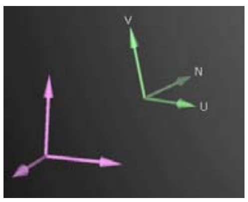
22

Parallel Projections

Lecture 4

Step2:

$$\vec{R_z} = \frac{\vec{VPN}}{\left|\vec{VPN}\right|} \qquad \vec{R_x} = \frac{\vec{VUP} \times \vec{R_z}}{\left|\vec{VUP} \times \vec{R_z}\right|} \qquad \vec{R_y} = \vec{R_z} \times \vec{R_x}$$



$$R = \begin{bmatrix} R_{1x} & R_{2x} & R_{3x} & 0 \\ R_{1y} & R_{2y} & R_{3y} & 0 \\ R_{1z} & R_{2z} & R_{3z} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

23

Parallel Projections

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At this point we are already in Eye Coordinates

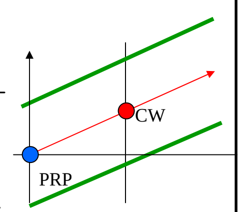
Step3:

$$CW = \begin{bmatrix} \frac{u_{\max}+u_{\min}}{2} \\ \frac{v_{\max}+v_{\min}}{2} \\ 0 \\ 1 \end{bmatrix} \qquad PRP = \begin{bmatrix} prp_u \\ prp_v \\ prp_n \\ 1 \end{bmatrix} \qquad DOP = \begin{bmatrix} dop_x \\ dop_y \\ dop_z \\ 0 \end{bmatrix} = CW - PRP$$

$$\begin{bmatrix} 1 & 0 & shear_x & 0 \\ 0 & 1 & shear_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$shear_x = -\frac{dop_x}{dop_z}$$

$$shear_y = -\frac{dop_y}{dop_z}$$



The shear is zero in the simple case: CW=(0,0,*) and PRP=(0,0,*)

24

Parallel Projections

Lecture 4

Step4:

Translation

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{u_{max}+u_{min}}{2} \\ 0 & 1 & 0 & -\frac{v_{max}+v_{min}}{2} \\ 0 & 0 & 1 & -F \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} \frac{2}{u_{max}-u_{min}} & 0 & 0 & 0 \\ 0 & \frac{2}{v_{max}-v_{min}} & 0 & 0 \\ 0 & 0 & \frac{1}{F-B} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

25

Perspective Projections

Lecture 4

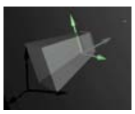
1. Translate VRP to the origin

2. Rotate VRC such that n axis (VPN) becomes z, u axis becomes x, and v axis becomes y.

3. Translate such that the center of Projection (COP), given by the PRP, is at the origin.

4. Shear such that the center line of the view volume becomes the z axis.

5. Scale such that the view volume becomes the canonical perspective view volume, the truncated right pyramid defined by the six planes



26

Perspective Projections

Step1:

Simply the negative of the VRP vector

$$\begin{bmatrix} 1 & 0 & 0 & -vrp_x \\ 0 & 1 & 0 & -vrp_y \\ 0 & 0 & 1 & -vrp_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step2:

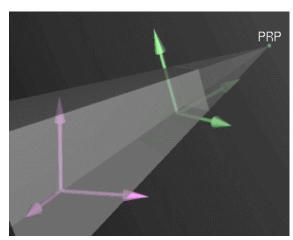
$$R_z = \frac{VPN}{|VPN|} \qquad R_x = \frac{VUP \times R_z}{|VUP \times R_z|} \qquad R_y = R_z \times R_x$$

$$R = \begin{bmatrix} R_{1x} & R_{2x} & R_{3x} & 0 \\ R_{1y} & R_{2y} & R_{3y} & 0 \\ R_{1z} & R_{2z} & R_{3z} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Perspective Projections

Step3:

$$\begin{bmatrix} 1 & 0 & 0 & prp_u \\ 0 & 1 & 0 & prp_v \\ 0 & 0 & 1 & prp_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



PRP becomes the origin

Perspective Projections

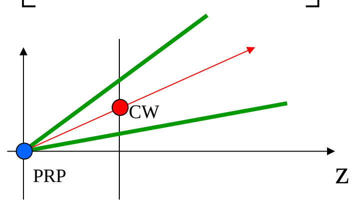
Lecture 4

Step4:

$$\begin{bmatrix} 1 & 0 & shear_x & 0 \\ 0 & 1 & shear_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$shear_x = -\frac{dop_x}{dop_z}$$

$$shear_y = -\frac{dop_y}{dop_z}$$



29

Perspective Projections

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Step5:

$$VRP' = S_{step4} \bullet T_{step3} \bullet \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{2vrp'_z}{(u_{\max}-u_{\min})(vrp'_z+B)} & 0 & 0 & 0 \\ 0 & \frac{2vrp'_z}{(v_{\max}-v_{\min})(vrp'_z+B)} & 0 & 0 \\ 0 & 0 & \frac{-1}{vrp'_z-B} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

30

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Perspective Projections

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$$\begin{bmatrix} \text{Othrographic} \\ \text{Projection} \\ \text{Matrix} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -D \\ 0 & 0 & \left(\frac{1}{D}\right) & 0 \end{bmatrix} \qquad Where: D = \cot\left(\frac{fov}{2}\right)$$

$$\begin{bmatrix} x \\ y \\ (z - D) \\ \left(\frac{z}{D}\right) \end{bmatrix} \Rightarrow \begin{bmatrix} D\frac{x}{z} & D\frac{y}{z} & D\frac{z-D}{z} \end{bmatrix} = \begin{bmatrix} X & Y & Z \end{bmatrix}$$

31

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Another way to look at it

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- Canonical view volume
  - OpenGL
    - Cube from (-1,-1,-1) to (1,1,1)
    - Near plane rectangle
      (left, bottom, -near) becomes (-1,-1,-1)
      (right, top, -near) becomes (1,1,-1)
  - Far plane maps onto z=1

$$\begin{pmatrix} \frac{2\,near}{right-left} & 0 & A & 0 \\ 0 & \frac{2\,near}{top-bottom} & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$A = \frac{right+left}{right-left}$$

$$B = \frac{top+bottom}{top-bottom}$$

$$C = -\frac{far+near}{far-near}$$

$$D = -\frac{2\,far\,near}{far-near}$$

0<near<far
BUT corresponds to negative z-axis

32