Modeling

Modeling - Numeric Representation of a Physical Object or Process

- Geometric Modeling
  - 3-D Coordinate Description of Objects

- Physics Modeling
  - Illumination (Light / Color)
  - Reflection (EM Scattering)
  - Interacting Forces (Jell-O)

- Motion Modeling
  - Animation
  - Camera Location

Modeling

- Graphics Pipeline

Data Objects → Conversion to Pixel Values → Display Device

- Geometric
- Vector Fields
- Character

- Projection
- Illumination
- Shading

- Deflect Beam
- Active Phosphor
Modeling

- Methods of Geometric Modeling
  - Boundary Representation (B-rep)
    - Interior / Exterior Interface
  - Spatial Representation
    - Interior Volume Cubes
- Space vs. Time Trade-off
- Development Tools
  - CAD Packages
  - User Applications
- Standards / Formats ???
  - PHIGS
  - Storage
  - Processing

Boundary Representation

- Surface Modeling
  - Quadratic Surfaces
  - Polygons / Facets
  - Splines
- Solid Modeling
  - Boolean Operators on Geometric Primitives
Quadratic Surfaces

• Exact Representation of Surface

• Limited Number of Shapes

\[ f(x, y, z) = Ax^2 + By^2 + Cz^2 + 2Dxy + 2Eyz + 2Fxz + 2Gx + 2Hy + 2Jz + K = 0 \]

Sphere: \( A = B = C = 1 \)  
\( K = -r^2 \)  
Plane: A thru \( F = 0 \)  
All others = 0

Quadratic Surfaces

• Matrix Formulation

\[
\begin{bmatrix}
A & D & F & G \\
D & B & E & H \\
F & E & C & J \\
G & H & J & K
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} = f(x, y, z) = [\mathbf{P}]^T [\mathbf{Q}][\mathbf{P}]
\]

• Operate on \([\mathbf{Q}]\) with 4x4 Transformation Matrices

• Compute Surface Normal \([\mathbf{N}] = [\partial f/\partial x \quad \partial f/\partial y \quad \partial f/\partial z] \]

• Test a Point wrt the Surface
Quadratic Surfaces

Polygons

- Good Representation for Planar Surfaces
- Approximation of a Curved Surface with Piecewise Linear Segments
- Smaller/More Facets Give Closer Approximation
- Special Hardware Provides Rapid Rendering
- Tessellation – Convert Forms into Polygons
Polygon Data

- Polygon Data Files
  - Explicit Polygon Vertices
    - Difficult Interaction
    - Redundant
    
    \[ P_1 = \{(V_{i1}, V_{i2}, V_{i3}), (V_{i4}, V_{i5}, V_{i6}), \ldots, (V_{in}, V_{in+1}, V_{in+2})\} \]
    \[ P_2 = \{(V_{m1}, V_{m2}, V_{m3}), (V_{m4}, V_{m5}, V_{m6}), \ldots, (V_{mn}, V_{mn+1}, V_{mn+2})\} \]

- Vertex Table
  - Number of Vertices
  - Polygon Vert’s CCW

- Edge Table
  - Shared Edges

- Ancillary Data
  - Normal Vectors
  - Material Type
  - Shading Coefficients
  - Transparency

Polygon Error Checking

- Every Vertex is an End Point for at Least Two Edges
- Every Edge is a Part of at Least One Polygon
- Every Polygon is Closed
- Every Polygon Has at Least One Shared Edge

Waterproof Models
Polygon WireFrames

- Triangular Mesh
  - Guaranteed Coplanar Points
  - Compute Normal with Cross Product
- Quadrilateral Mesh
  - Points May Not Be Planar
  - Approximate Normal
  - OR
  - Convert to Triangular
- Fan Mesh
  - Common Center point
  - Guaranteed Coplanar Points

Polygon Mesh
Polygon Normals

Polygons Vertices in CCW Order when viewed from Outside

\[ \vec{V}_1 = (P_2 - P_0, P_3 - P_0, P_0 - P_0) \]

\[ \vec{V}_2 = (P_0 - P_0, P_3 - P_0, P_0 - P_0) \]

\[ \vec{N} = \vec{V}_1 \times \vec{V}_2 \]

Plane Eqn :

\[ Ax + By + Cz + D = 0 \]

\[
\begin{bmatrix}
A \\
B \\
C \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix}
\]

\[ D = -\langle \vec{N}, \vec{P} \rangle \]

\[ P \text{ is any point on the plane} \]

Ax + By + Cz + D > 0

[P] is above the plane (Outside the surface)

Ax + By + Cz + D < 0

[P] is below the plane (Inside the surface)
Solid Modeling

• CSG - Constructive Solid Geometry

• Widely Used in CAD/CAM Packages
  • Object modeling for
    • Casting
    • Machining
    • Extruding

B-Rep of a Box

Solid Model of a Box

8 Vertices $\rightarrow$ 24 floats
12 Edges
6 Polygon faces

Vectors for the Origin, Length, Width, and Height
4 Vectors $\rightarrow$ 12 floats

Solid Modeling

• Set of Geometric Primitives
  • Blocks
  • Pyramids
  • Cylinders
  • Spheres
  • etc.

• Combine Primitives with Boolean Operators
  • Union $A \cup B$
  • Intersection $A \cap B$
  • Difference $A - B$

A
B

$A \cup B$  $A \cap B$  $A - B$
Facet Modeling

Solid Modeling

Drawbacks

- Difficult to model arbitrary surface
- Increased computation when rendering