Interactive Computer Graphics

Lecture 3

Aliasing and Anti-Aliasing

- Hearn & Baker, chapter 4-17

2D transforms

- Hearn & Baker, chapter 5

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Aliasing and Anti-Aliasing

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Problem: “jaggies”

Also known as “aliasing.” It results from sampling a pattern (signal) at limited resolution.

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Aliasing and Anti-Aliasing -Examples

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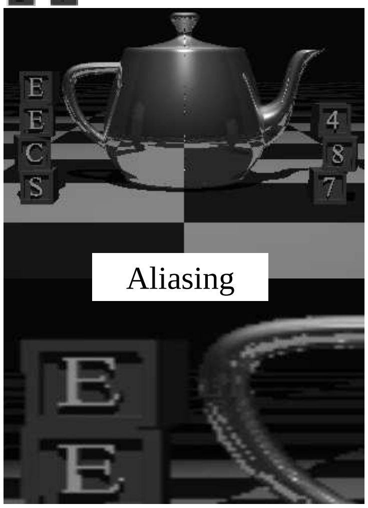

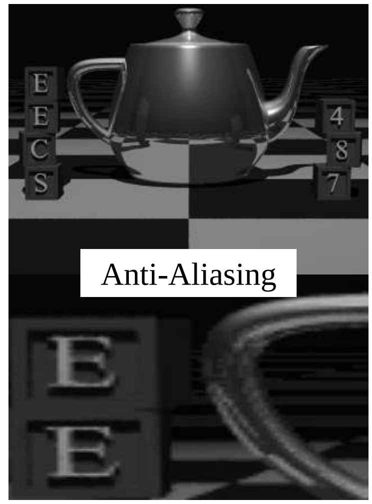

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Aliasing in photos

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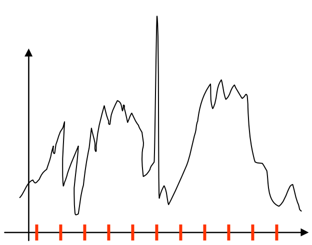
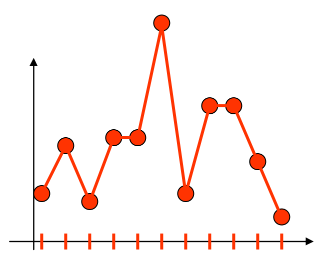
- A piece of practical knowledge
  - Example web page
  - Often better to blur before resizing





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Aliasing and Anti-Aliasing

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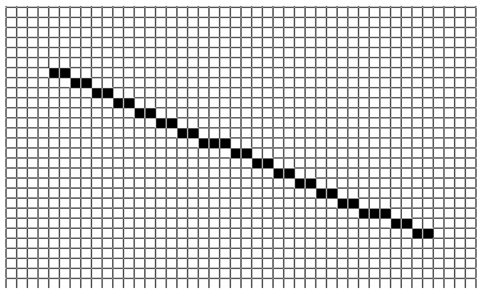
To fully understand the causes of aliasing requires the use of Signal Processing and Fourier Transforms.

We can look at Aliasing from a more intuitive basis.



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Aliasing and Anti-Aliasing

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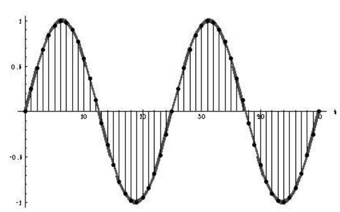
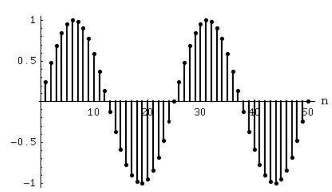
Effects of Sampling





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Aliasing and Anti-Aliasing

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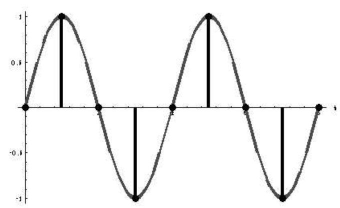
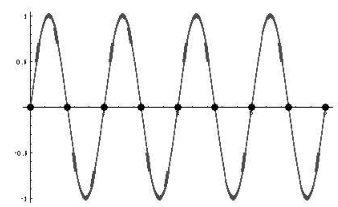
Effects of Sampling





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Aliasing and Anti-Aliasing

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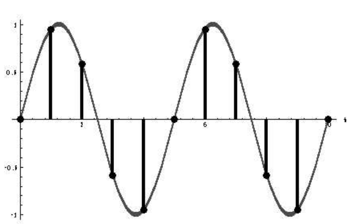
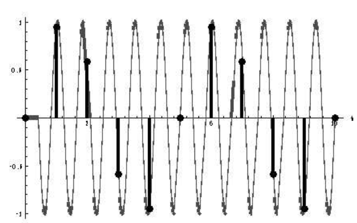
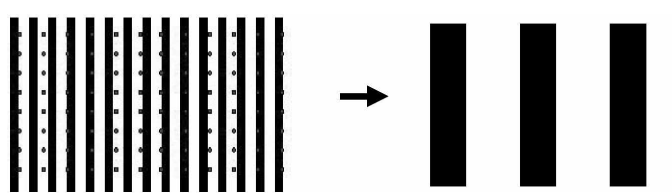
Aliasing







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Aliasing and Anti-Aliasing

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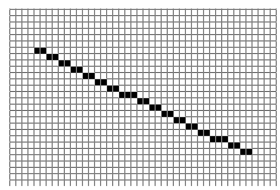
Anti-Aliasing is the process that attempts to prevent or fix the problem.

- More Resolution
- Unweighted Area Sampling
- Weighted Area Sampling
- Post image creation filtering



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Aliasing and Anti-Aliasing

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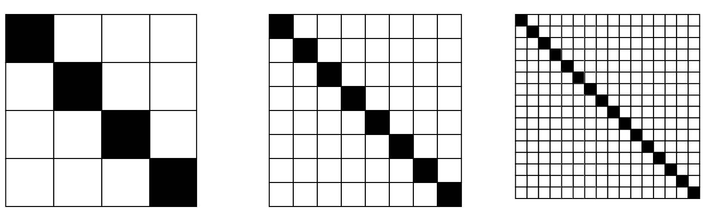
More Resolution



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Aliasing and Anti-Aliasing

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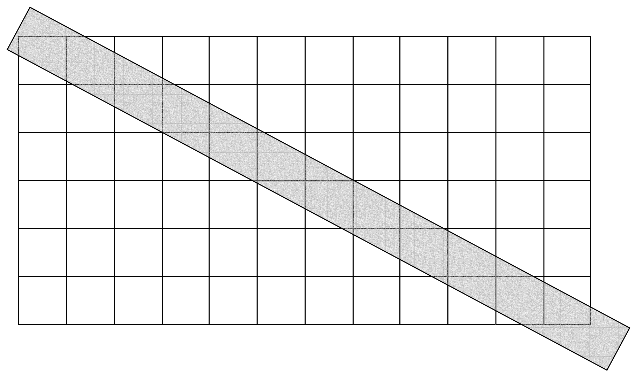
Unweighted Area Sampling



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Aliasing and Anti-Aliasing

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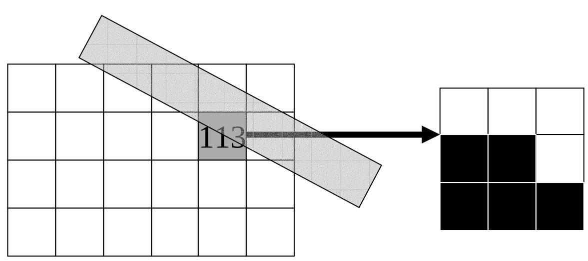
Unweighted Area Sampling



Pixel

Sub Pixels

255 * 4/9 = 113

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Aliasing and Anti-Aliasing

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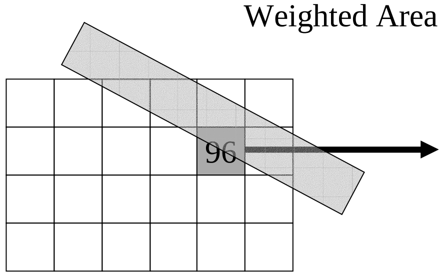
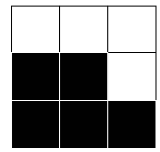
Weighted Area Sampling



Pixel



Sub Pixels

1	2	1
2	4	2
1	2	1

Sub Pixels Weights

255 * (1+2+1+2)/16 = 96

Aliasing and Anti-Aliasing

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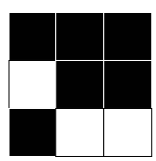
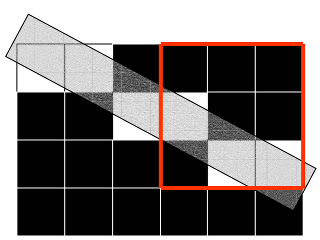
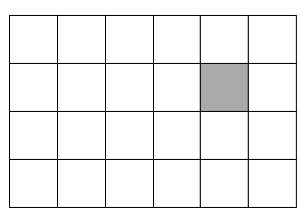
Post image creation filtering



= 3/9 * 255 = 85





Aliasing and Anti-Aliasing

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Post-image processing:
- fewer samples (less work)
- more blur (worse quality)

Weighted area sampling:
- slightly less blurry than unweighted

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Matrix Transformations

Lecture 3

Basic Graphics Transforms
- Translation
- Scaling
- Rotation
- Reflection
- Shear

All Can be Expressed As Linear
Functions of the Original Coordinates :

$$x' = Ax + By + C$$
$$y' = Dx + Ey + F$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A & B & C \\ D & E & F \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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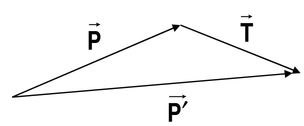
Affine Transformations

- Preserve Parallel Lines
- Finite Points Map to Finite Points
- Translation, Rotation, and Reflection Preserve :
  - Angles
  - Lengths

2-D Transformations - Translation

$$x' = x + T_x$$
$$y' = y + T_y$$



Vectors : $\vec{P}' = \vec{P} + \vec{T}$

Matrices : $[P'] = [P] + [T]$

$$\begin{bmatrix} P'_x \\ P'_y \end{bmatrix} = \begin{bmatrix} P_x \\ P_y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix} = \begin{bmatrix} P_x + T_x \\ P_y + T_y \end{bmatrix}$$

Transform Polygons by Transforming Each Vertex

2-D Transformations - Scaling

Lecture 3

$$x' = S_x \cdot x$$
$$y' = S_y \cdot y$$

$$[\mathbf{P}'] = [\mathbf{S}][\mathbf{P}]$$

$$\begin{bmatrix} P'_x \\ P'_y \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} P_x \\ P_y \end{bmatrix} = \begin{bmatrix} S_x \cdot P_x \\ S_y \cdot P_y \end{bmatrix}$$

Uniform Scaling : $S_x = S_y$

Differential Scaling : $S_x \neq S_y$

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2-D Transformations - Rotation

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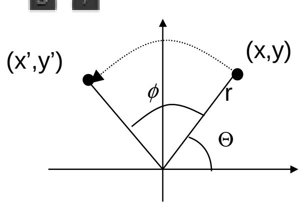

Rotation about the Origin

$$r\cos\Theta = x$$
$$r\sin\Theta = y$$

$$x' = r\cos(\Theta+\phi) = r\cos\Theta\cos\phi - r\sin\Theta\sin\phi$$
$$y' = r\sin(\Theta+\phi) = r\cos\Theta\sin\phi + r\sin\Theta\cos\phi$$

$$x' = x\cos\phi - y\sin\phi$$
$$y' = x\sin\phi + y\cos\phi$$

$$[\mathbf{P}'] = [\mathbf{R}(\phi)][\mathbf{P}]$$

$$\begin{bmatrix} P'_x \\ P'_y \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} P_x \\ P_y \end{bmatrix}$$

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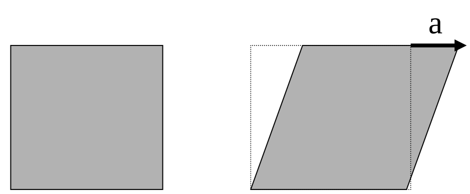
Shear Transforms

Shear: An action or stress resulting from applied forces that causes two contiguous parts of a body to slide relatively to each other.



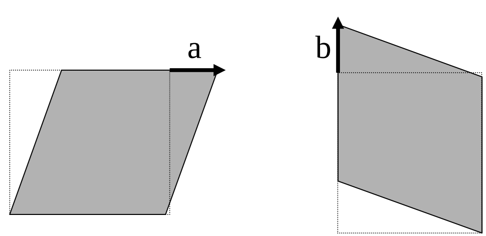
Shear Transforms

$$\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$$

Shear Translation in X

$$\begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}$$

Shear Translation in Y



Shear Transforms

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Properties:
- It is an Affine Transform
- a and b are the proportionality constant

A shear transform application:
- Fast and efficient rotation
- It will be used in perspective transformations

Shear is easily invertable

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Shear Transforms

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Shear rotation

$$\begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \beta & 1 \end{bmatrix}\begin{bmatrix} 1 & \lambda \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\alpha = -\tan\left(\tfrac{\theta}{2}\right)$$

$$\beta = \sin\theta$$

$$\lambda = -\tan\left(\tfrac{\theta}{2}\right)$$

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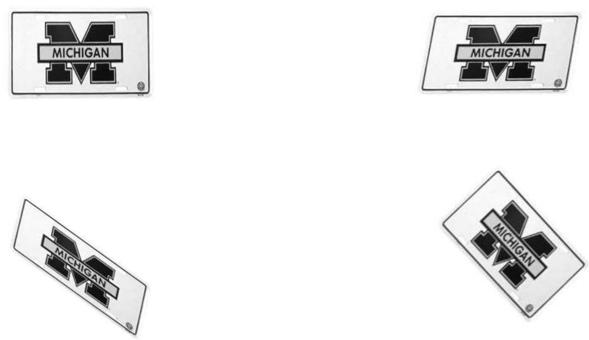
Shear Raster Rotation - Example



Reflection Transforms

Reflection is a transform that allows vectors to be flipped about an axis as if reflected in a mirror.

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Reflection About YZ　　Reflection About XZ　　Reflection About XY

2-D Transformations

Lecture 3

- Unify Transformation Operations
  - All Transformations Represented as Matrix Products

$$[\mathbf{T}] \equiv \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \quad [\mathbf{S}] \equiv \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad [\mathbf{R}(\phi)] \equiv \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[\mathbf{P}'] = [\mathbf{T}][\mathbf{P}] \qquad [\mathbf{P}'] = [\mathbf{S}][\mathbf{P}] \qquad [\mathbf{P}'] = [\mathbf{R}(\phi)][\mathbf{P}]$$

Homogeneous Coordinates

$$[\mathbf{P}] = \begin{bmatrix} P_x \\ P_y \\ 1 \end{bmatrix}$$

← **NOT** the z coordinate

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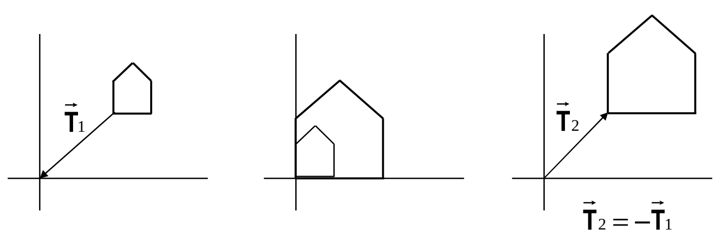
Transform Combinations

Lecture 3

- Scaling About a Point
  - Translate to Origin
  - Apply Scale Matrix
  - Translate Back to Original Position

$$[\mathbf{P}'] = [\mathbf{T}_2][\mathbf{S}][\mathbf{T}_1][\mathbf{P}]$$



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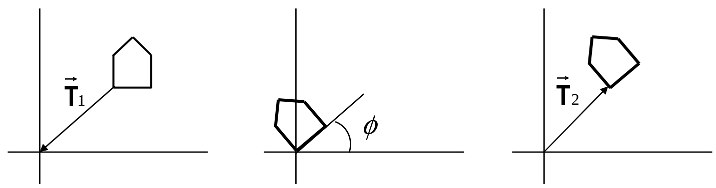
Transform Combinations

Lecture 3

- Rotation About a Point
  - Translate to Origin
  - Apply Rotation Matrix
  - Translate Back to Original Position

$$[P'] = [T_2][R(\phi)][T_1][P]$$



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Homogeneous coordinates in 2D

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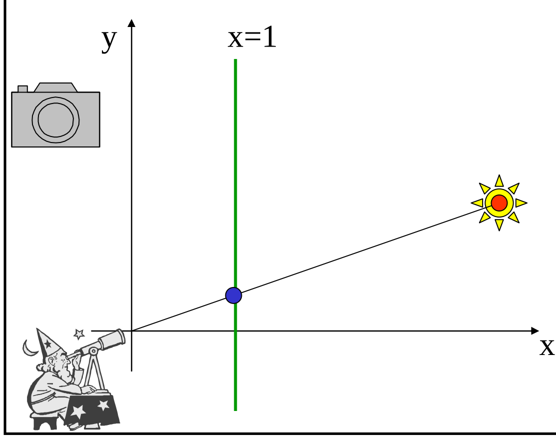
- How to represent perspective projection
  - It is hard!



Given the coordinates of the orange point find the coordinates of the blue point

Points: $(x,y) \rightarrow (x,y,1)$

Equivalency:
$(x,y,w) \equiv (x/a,\ y/a,\ w/a)$

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An arbitrary camera?

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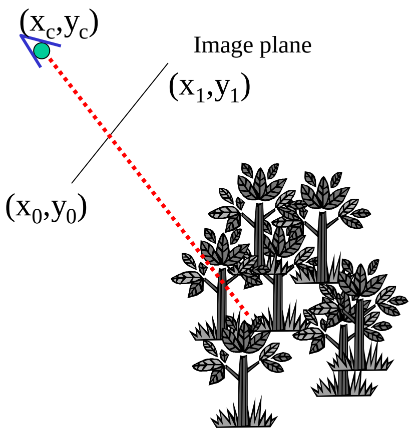
- Need some systematic way of doing this



Approach:

One thing at a time

- First transform everything so that the camera is in the canonic position
  - Matrix M
- Then perform projection
  - Matrix P
- Combine the result P*M

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