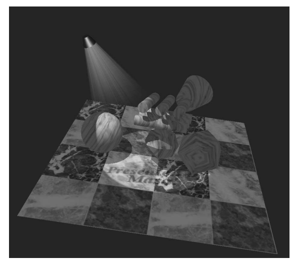
Transforms II

Lecture 4

Overview

- Homogeneous Coordinates
- 3-D Transforms
- Viewing Projections



Homogeneous Coordinates

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$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \\ \frac{z}{w} \end{bmatrix} \qquad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \quad \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \\ \frac{z}{w} \\ 1 \end{bmatrix}$$

Allows translations to be included into matrix transform.

Allows us to distinguish between a vector and a point.

In perspective transformations the extra coordinate can be thought to contain the perspective information or scaling.

Homogeneous Coordinates

Lecture 4

$$\begin{bmatrix}1\\0\\0\\1\end{bmatrix} \longrightarrow \begin{bmatrix}1\\0\\0\\w\end{bmatrix} \longrightarrow \begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

- As w gets smaller Real space X gets Larger
- When w reaches 0 X is now at infinity
- Homogeneous coordinates allows us to deal mathematically with infinity

3

Homogeneous Coordinates

Lecture 4

$$\begin{bmatrix}\frac{x}{w}\\\frac{y}{w}\\\frac{z}{w}\\1\end{bmatrix} \qquad \begin{bmatrix}x\\y\\z\\1\end{bmatrix} \qquad \begin{bmatrix}x\\y\\z\\0\end{bmatrix}$$

Point Vector

Many different homogeneous space points for the same real space point

Vectors can not be translated

Points can be translated and blended

Exercise: determine legal combinations

4

3-D Transforms

Lecture 4

Rotation

$$\begin{bmatrix} x''' \\ y''' \\ z''' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ z'' \\ 1 \end{bmatrix}$$

Rotation about X

$$\begin{bmatrix} x'' \\ y'' \\ z'' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & \sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$

Rotation about Y

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\lambda & -\sin\lambda & 0 & 0 \\ \sin\lambda & \cos\lambda & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotation about Z

5

3-D Transforms

Lecture 4

Rotation

$$\begin{bmatrix} x''' \\ y''' \\ z''' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & \sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\lambda & -\sin\lambda & 0 & 0 \\ \sin\lambda & \cos\lambda & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

6

3-D Transforms

Lecture 4

Scale

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} scalex & 0 & 0 & 0 \\ 0 & scaley & 0 & 0 \\ 0 & 0 & scalez & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

7

3-D Transforms

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Translation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

8

3-D Transforms

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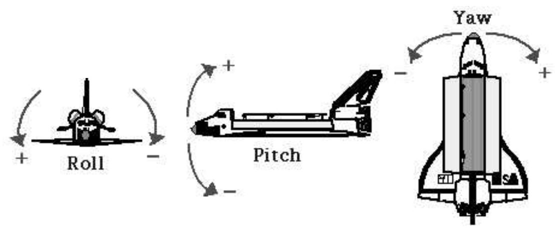
Composite Rotation Matrix

$$[R] = [Roll][Pitch][Yaw]$$



$$[M] = [R][S][T]$$

9

Transform composition

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- Ship moves
- Character moves
- All represented as a sequence of matrix transforms (dependent on time)

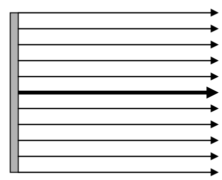
10

Viewing Projections

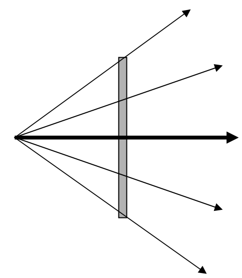
Lecture 4

How do we see in 3-D

Parallel Projections



Perspective Projections



11

Viewing Projections

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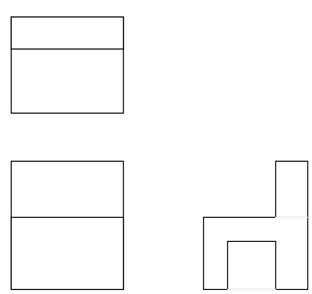
Parallel Projections

Elevations:

Projection plane is perpendicular to a principle axis. Front, Top (Plan), Side.



12

Viewing Projections

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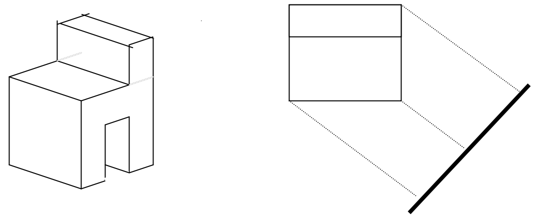
Parallel Projections

Axonometric:
Projection plane is not orthogonal to a principle axis.

Isometric:
Direction of projection makes equal angles with each principal axis.



13

Viewing Projections

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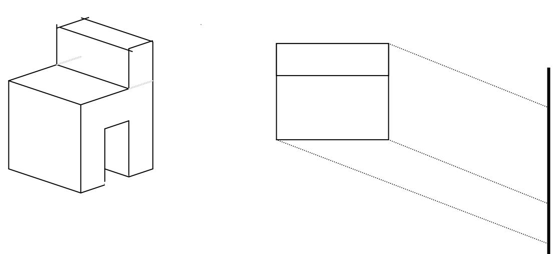
Parallel Projections

Oblique:
Direction of projection is not orthogonal to the projection plane.



14

Viewing Projections

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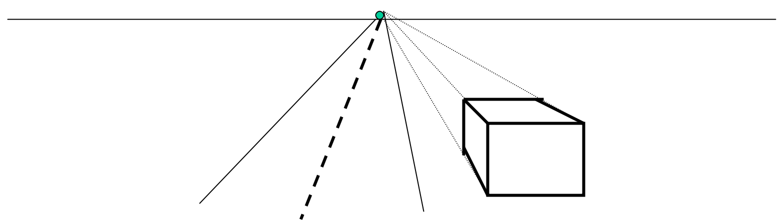
Perspective Projection

One-point:

One principle axis cut by projection plane.
One axis vanishing point.



15

Viewing Projections

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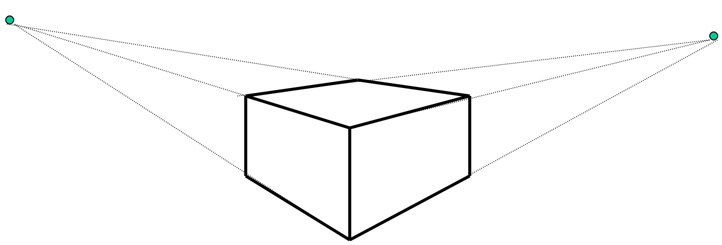
Perspective Projection

Two-point:

Two principle axes cut by projection plane.
Two vanishing points.



16

Viewing Projections

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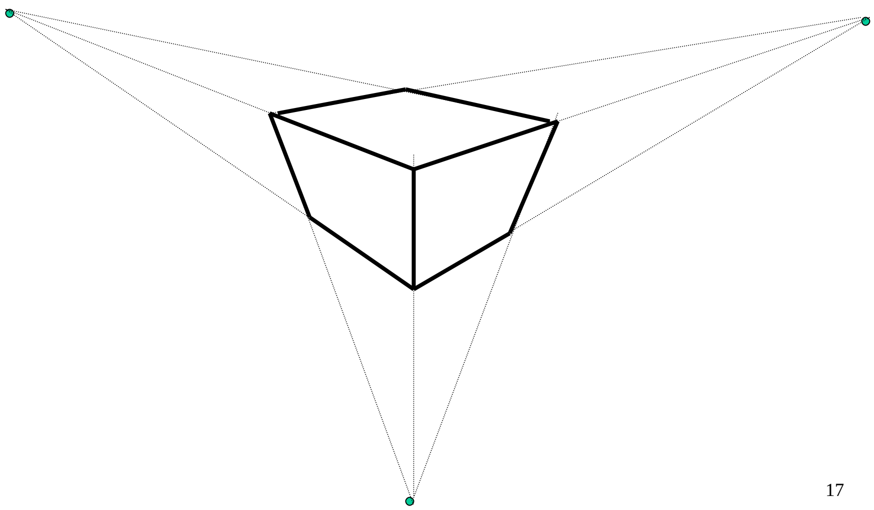
Perspective Projection Three-point



17

Viewing Projections

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Definitions

VRC	-	Viewing Reference Coordinate (system)
VRP	-	View Reference Point
VPN	-	View Plane Normal
VUP	-	View Up Direction
DOP	-	Direction of Projection
PRP	-	Projection Reference Point (EYE) Center of Projection
VP	-	Viewing Plane
BCP	-	Back Clipping Plane
FCP	-	Front Clipping Plane

18

Notation

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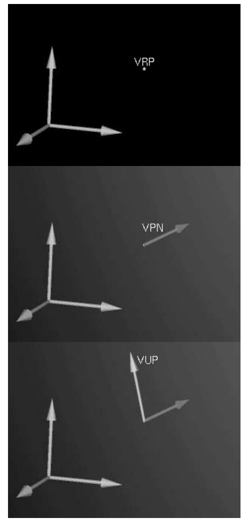
- VRP – origin of VRC
- VPN – view plane normal
- VUP – view up direction



http://www.scs.leeds.ac.uk/cuddles/hyperbks/Rendering/Pipeline/view_def_vref.html

19

Parallel Projections

Lecture 4

1. Translate VRP to the origin

2. Rotate VRC such that n axis (VPN) becomes z, u axis becomes x, and v axis becomes y.

3. Shear such that the direction of projection becomes parallel to the z axis.

4. Translate and scale into the parallel-projection canonical view volume.

20

Parallel Projections

Lecture 4

Step1:

Simply the negative of the VRP vector

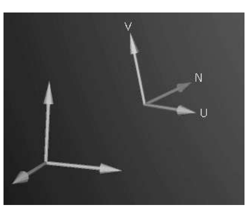
$$\begin{bmatrix} 1 & 0 & 0 & -vrp_x \\ 0 & 1 & 0 & -vrp_y \\ 0 & 0 & 1 & -vrp_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Parallel Projections

Lecture 4

Step2:

$$\overrightarrow{R_z} = \frac{\overrightarrow{VPN}}{\left|\overrightarrow{VPN}\right|} \qquad \overrightarrow{R_x} = \frac{\overrightarrow{VUP} \times \overrightarrow{R_z}}{\left|\overrightarrow{VUP} \times \overrightarrow{R_z}\right|} \qquad \overrightarrow{R_y} = \overrightarrow{R_z} \times \overrightarrow{R_x}$$



$$R = \begin{bmatrix} R_{1x} & R_{2x} & R_{3x} & 0 \\ R_{1y} & R_{2y} & R_{3y} & 0 \\ R_{1z} & R_{2z} & R_{3z} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Parallel Projections

Lecture 4

Step3:

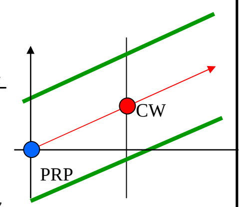
At this point we are already in Eye Coordinates

$$CW = \begin{bmatrix} \frac{u_{\max}+u_{\min}}{2} \\ \frac{v_{\max}+v_{\min}}{2} \\ 0 \\ 1 \end{bmatrix} \qquad PRP = \begin{bmatrix} prp_u \\ prp_v \\ prp_n \\ 1 \end{bmatrix} \qquad DOP = \begin{bmatrix} dop_x \\ dop_y \\ dop_z \\ 0 \end{bmatrix} = CW - PRP$$

$$\begin{bmatrix} 1 & 0 & shear_x & 0 \\ 0 & 1 & shear_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$shear_x = -\frac{dop_x}{dop_z}$$

$$shear_y = -\frac{dop_y}{dop_z}$$



The shear is zero in the simple case: CW=(0,0,*) and PRP=(0,0,*)

23

Parallel Projections

Lecture 4

Step4:

Translation

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{u_{\max}+u_{\min}}{2} \\ 0 & 1 & 0 & -\frac{v_{\max}+v_{\min}}{2} \\ 0 & 0 & 1 & -F \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} \frac{2}{u_{\max}-u_{\min}} & 0 & 0 & 0 \\ 0 & \frac{2}{v_{\max}-v_{\min}} & 0 & 0 \\ 0 & 0 & \frac{1}{F-B} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

24

Parallel Projections

Lecture 4

1. Translate VRP to the origin

2. Rotate VRC: n → z, u → x, v → y.

3. Shear so DOP aligns with z.

4. Translate and scale into canonical view volume.

$P = ST_2SRT_1$

Q: How do we get the image?

25

Perspective Projections

Lecture 4

1. Translate VRP to the origin

2. Rotate VRC: n → z, u → x, v → y.

3. Translate PRP to origin

4. Shear so DOP aligns with z.

5. Scale to get canonical view volume:
   $x = z$, $z = -z$, $y = z$, $y = -z$, $z = z_{min}$, $z = -1$

26

Perspective Projections

Lecture 4

Step1:

Simply the negative of the VRP vector

$$\begin{bmatrix} 1 & 0 & 0 & -vrp_x \\ 0 & 1 & 0 & -vrp_y \\ 0 & 0 & 1 & -vrp_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step2:

$$R_z = \frac{VPN}{|VPN|} \qquad R_x = \frac{VUP \times R_z}{|VUP \times R_z|} \qquad R_y = R_z \times R_x$$

$$R = \begin{bmatrix} R_{1x} & R_{2x} & R_{3x} & 0 \\ R_{1y} & R_{2y} & R_{3y} & 0 \\ R_{1z} & R_{2z} & R_{3z} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

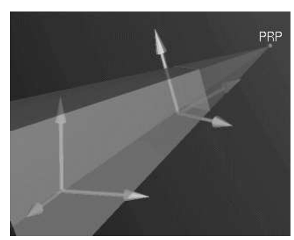
27

Perspective Projections

Lecture 4

Step3:

$$\begin{bmatrix} 1 & 0 & 0 & -prp_u \\ 0 & 1 & 0 & -prp_v \\ 0 & 0 & 1 & -prp_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Map PRP to the origin

28

Perspective Projections

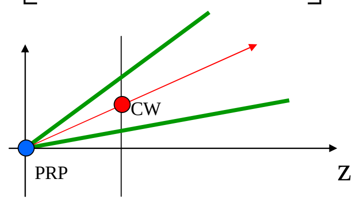
Lecture 4

Step4:

$$\begin{bmatrix} 1 & 0 & shear_x & 0 \\ 0 & 1 & shear_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$shear_x = -\frac{dop_x}{dop_z}$$

$$shear_y = -\frac{dop_y}{dop_z}$$



29

Perspective Projections

Lecture 4

Step5:

$$VRP' = S_{step4} \bullet T_{step3} \bullet \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{2vrp'_z}{(u_{\max}-u_{\min})(vrp'_z+B)} & 0 & 0 & 0 \\ 0 & \frac{2vrp'_z}{(v_{\max}-v_{\min})(vrp'_z+B)} & 0 & 0 \\ 0 & 0 & \frac{-1}{vrp'_z+B} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

30

Perspective Projections

Lecture 4

1. Translate VRP to the origin

2. Rotate VRC: n → z, u → x, v → y.

3. Translate PRP to origin

4. Shear so DOP aligns with z.

5. Scale to get canonical view volume:
   $x = z,\ z = -z,\ y = z,\ y = -z,\ z = z_{min},\ z = -1$

$P = SZT_2RT_1$

Q: How do we get the image?

31

Perspective Projections

Lecture 4

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix}$$

d is the distance to the front clipping plane

$$\begin{bmatrix} x \\ y \\ z \\ -z/d \end{bmatrix} \Rightarrow \begin{bmatrix} x\frac{-d}{z} & y\frac{-d}{z} & -d \end{bmatrix}$$

32

Another approach: OpenGL

- Canonical view volume
  - OpenGL
    - Cube from (-1,-1,-1) to (1,1,1)
    - Near plane rectangle
      (left, bottom, -near) becomes (-1,-1,-1)
      (right, top, -near) becomes (1,1,-1)
  - Far plane maps onto z = 1