Modeling

Lecture 5

Modeling - Numeric Representation of
a Physical Object or Process

- Geometric Modeling
  - 3-D Coordinate Description of Objects
- Physics Modeling
  - Illumination (Light / Color)
  - Reflection (EM Scattering)
  - Interacting Forces (Jell-O)
- Motion Modeling
  - Animation
  - Camera Location

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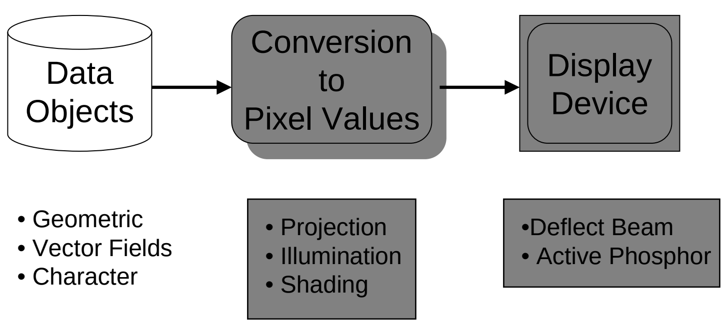
Modeling

Lecture 5

- Graphics Pipeline



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A ten minute project

Lecture 5

Describe this room and what's inside it

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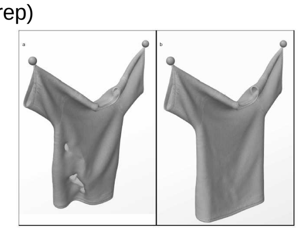
Modeling

Lecture 5

- Methods of Geometric Modeling
  - Boundary Representation (B-rep)
    - Interior / Exterior Interface
  - Spatial Representation
    - Interior Volume Cubes
- Space vs. Time Trade-off
- Development Tools
  - CAD Packages
  - User Applications
- Standards / Formats ????
  - PHIGS
  - Storage
  - Processing



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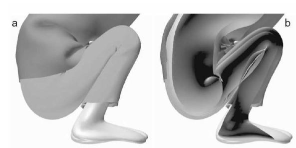

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Boundary Representation

Lecture 5

- Surface Modeling
  - Quadratic Surfaces
  - Polygons / Facets
  - Splines
- Solid Modeling
  - Boolean Operators on Geometric Primitives

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Quadratic Surfaces

Lecture 5

- Exact Representation of Surface
- Limited Number of Shapes

$$
\begin{aligned}
f(x, y, z) = Ax^2 + By^2 + Cz^2 \\
+ 2Dxy + 2Eyz + 2Fxz \\
+ 2Gx + 2Hy + 2Jz + K = 0
\end{aligned}
$$

Sphere : A = B = C = 1
K = $-r^2$
All others = 0

Plane : A thru F = 0

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Quadratic Surfaces

Lecture 5

- Matrix Formulation

$$[\mathbf{Q}] = \begin{bmatrix} A & D & F & G \\ D & B & E & H \\ F & E & C & J \\ G & H & J & K \end{bmatrix} \qquad [\mathbf{P}] = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$f(x,y,z) = [\mathbf{P}]^T \; [\mathbf{Q}][\mathbf{P}]$$

- Operate on [Q] with 4x4 Transformation Matrices
- Compute Surface Normal $[\mathbf{N}] = [\partial f/\partial x \quad \partial f/\partial y \quad \partial f/\partial z]$
- Test a Point wrt the Surface

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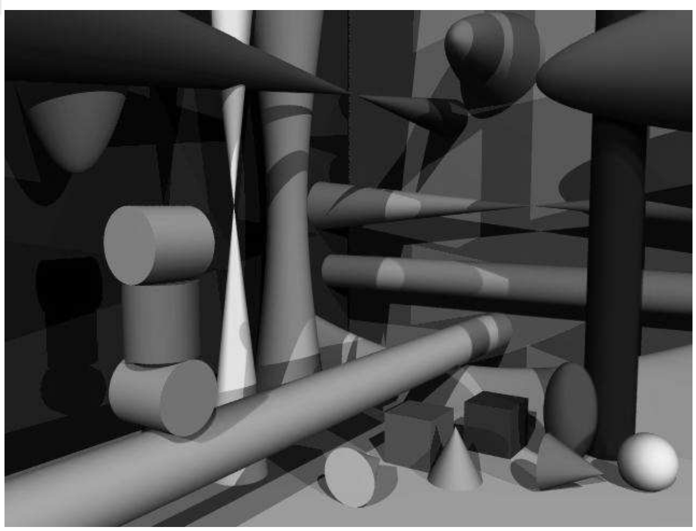
Quadratic Surfaces

Lecture 5



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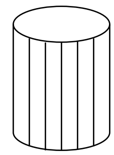
Polygons

Lecture 5



- Good Representation for Planar Surfaces
- Approximation of a Curved Surface with Piecewise Linear Segments
- Smaller/More Facets Give Closer Approximation
- Special Hardware Provides Rapid Rendering
- Tessellation – Convert Forms into Polygons

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Polygon Data

Lecture 5

- Polygon Data Files
  - Explicit Polygon Vertices
    - Difficult Interaction
    - Redundant

    Polygon soup

    $$P_1 = \{(V_{x1}, V_{y1}, V_{z1}), (V_{x2}, V_{y2}, V_{z2}), \ldots, (V_{xn}, V_{yn}, V_{zn})\}$$

    $$P_2 = \{(V_{xn+1}, V_{yn+1}, V_{zn+1}), \ldots, (V_{xn+m}, V_{yn+m}, V_{zn+m})\}$$

  - Vertex Table
    - Number of Vertices
    - Polygon Vert's CCW

    Polygonal mesh

    $$V = \{(x_1, y_1, z_1), (x_2, y_2, z_2), \ldots, (x_n, y_n, z_n)\}$$

    $$P_1 = \{1,3,6\} \qquad P_2 = \{2,4,5\}$$

  - Edge Table
    - Shared Edges

    Maybe even more general

    $$V = \{V_1, V_2, \ldots, V_n\}$$

    $$E_1 = \{V_1, V_2\} \quad E_2 = \{V_2, V_3\} \quad E_3 = \{V_3, V_1\}$$

    $$P_1 = \{E_1, E_2, E_3\} \qquad P_2 = \{E_3, E_4, E_5, E_6\}$$

  - Ancillary Data
    - Normal Vectors
    - Material Type
    - Shading Coefficients
    - Transparency

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Polygon Error Checking

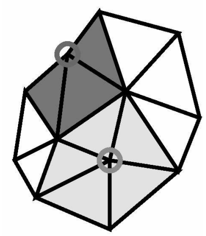
Lecture 5

- Every Vertex is an End Point for at Least Two Edges
- Every Edge is a Part of at Least One Polygon
- Every Polygon is Closed
- Every Polygon Has at Least One Shared Edge

Waterproof Models

Surface of a solid

Manifold meshes



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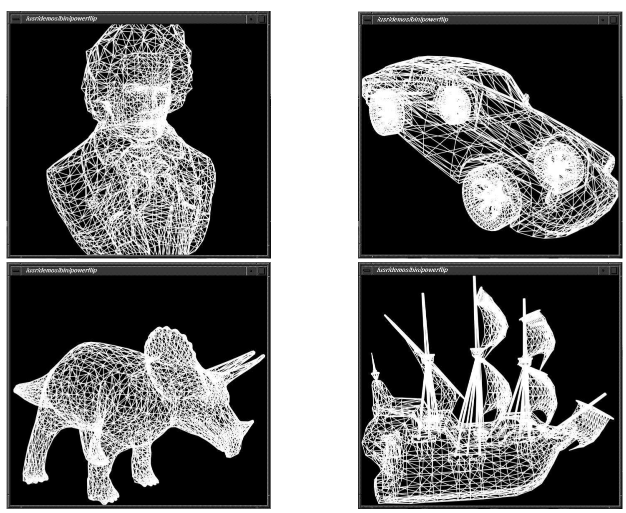
Polygon WireFrames

Lecture 5



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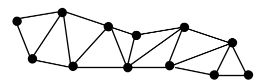
Polygon Mesh

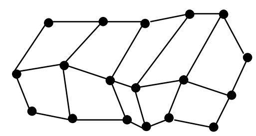
Lecture 5

- Triangular Mesh
  - Guaranteed Coplanar Points
  - Compute Normal with Cross Product

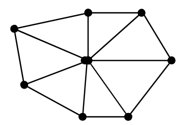


- Quadrilateral Mesh
  - Points May Not Be Planar
  - Approximate Normal

  OR
  - Convert to Triangular



- **Fan Mesh**
  - **Common Center point**
  - Guaranteed Coplanar Points



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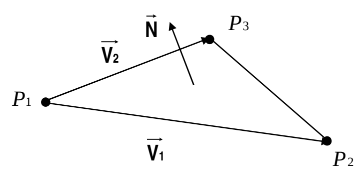
Polygon Normals

Lecture 5

Polygon Vertices in CCW Order when viewed from Outside



$$\vec{V_1} = \langle (P_{x2} - P_{x1}), (P_{y2} - P_{y1}), (P_{z2} - P_{z1}) \rangle$$

$$\vec{V_2} = \langle (P_{x3} - P_{x1}), (P_{y3} - P_{y1}), (P_{z3} - P_{z1}) \rangle$$

$$\vec{N} = \vec{V_1} \times \vec{V_2}$$

Plane Eqn :

$$Ax + By + Cz + D = 0$$

$$[N] = [A \quad B \quad C] \qquad [P] = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$D = -([N] \cdot [P_n])$$

P is any point on the plane

$$Ax + By + Cz + D > 0$$

[P] is above the plane (Outside the surface)

$$Ax + By + Cz + D < 0$$

[P] is below the plane (Inside the surface)

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Polygon Normals

Lecture 5

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Question

Lecture 5

- Where do we get our meshes?
- How many triangles do they contain?
- Why do we need triangles, can we just use points?

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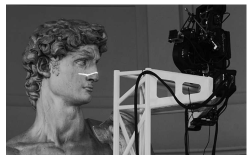
Digital Michelangelo

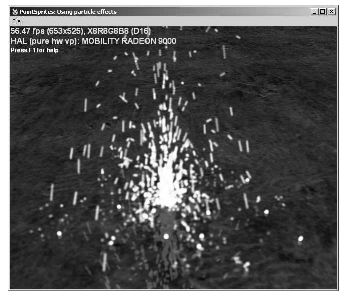
Lecture 5



Digital Michelangelo from Stanford

2 billion points

Mesh simplification
Progressive meshes
Compression
Skinning



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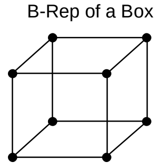
Solid Modeling

Lecture 5

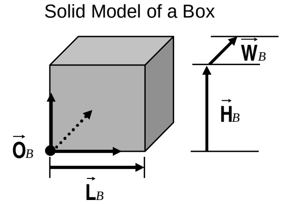
- CSG - Constructive Solid Geometry
- Widely Used in CAD/CAM Packages
  - Object modeling for
    - Casting
    - Machining
    - Extruding

B-Rep of a Box



8 Vertices ==> 24 floats
12 Edges
6 Polygon faces

Solid Model of a Box



Vectors for the Origin, Length, Width, and Height
4 Vectors ==> 12 floats

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Solid Modeling

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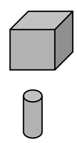
CSG: Constructive Solid Geometry

- Set of Geometric Primitives
  - Blocks
  - Pyramids
  - Cylinders
  - Spheres
  - etc.
- Combine Primitives with Boolean Operators
  - Union $A \cup B$ (A OR B)
  - Intersection $A \cap B$ (A AND B)
  - Difference $A - B$ (A Minus B)

A

B



$A \bigcup B$ $\quad A \bigcap B$ $\quad A - B$

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Solid Modeling

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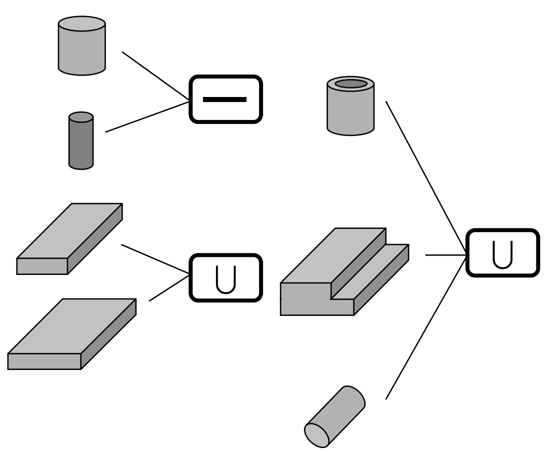
CSG: Constructive Solid Geometry



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Solid Modeling

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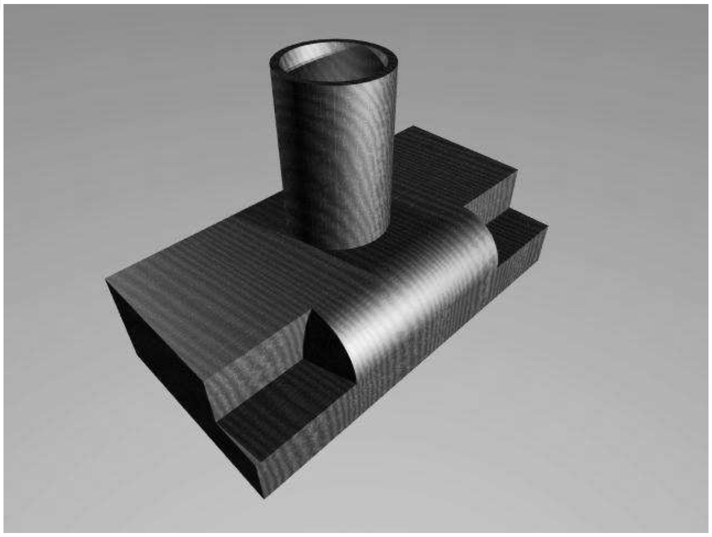

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Facet Modeling

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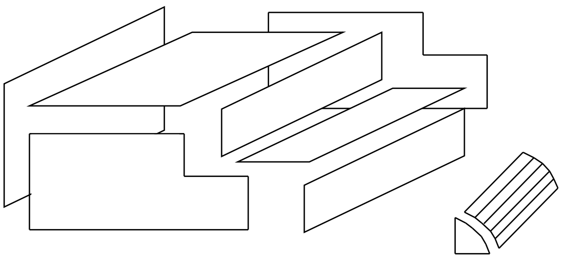
Frankly speaking,
it is a hard problem



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Solid Modeling

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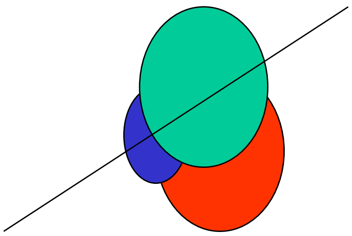
Drawbacks

- Difficult to model arbitrary surface
- Increased computation when rendering

But when raytracing CSG works very well actually!



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Solid Modeling

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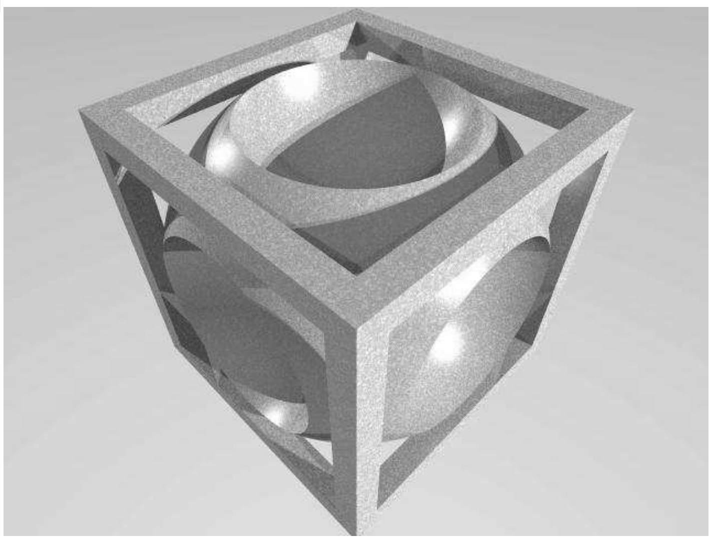

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Volumes

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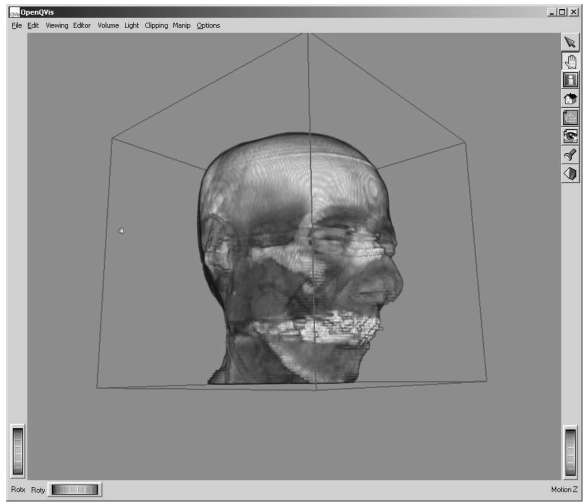
N*N*N



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