EECS 487, Fall 2005
Exam 2

December 21, 2005

This is a closed-book exam. Notes are not permitted.
Basic calculators are permitted, but not needed.
Explain or show your work for each question.

Name: ______________________________

uniqname: ______________________________

Honor code pledge: I have neither given nor received aid on this exam.

Signature: ______________________________

problem	***points***
1.	/16
2.	/16
3.	/14
4.	/14
5.	/16
Total	/76

1. Viewing (16 points)

Consider the perspective view volume (a truncated pyramid) defined in eye coordinates as follows. Eye coordinates are defined as in our textbook: by an eye location **e** at the apex of the pyramid, and orthonormal directions **u**, **v**, **w**. Let *n* and *f* be the *w*-coordinates of the near and far planes, let *r* and *l* be the *u*-coordinates of the right and left sides of the view volume in the near plane, and let *t* and *b* be the *v*-coordinates of its top and bottom sides in the near plane. (This is the same notation as used in our textbook.)

Recall the perspective matrix:

$$P = \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -nf \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(a) Express how P transforms an arbitrary point $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

(b) Where does P map $\begin{pmatrix} r \\ t \\ n \end{pmatrix}$

(c) Where does P map $\begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$

(d) What are the coordinates of the top right corner of the view volume in the *far* plane, before and after transformation by P? (Remember, the view volume is a truncated pyramid, not a box!)

2. Splines (16 points)

(a) Suppose we want to define a degree-2 polynomial curve $\mathbf{f}(t)$, parametrized over t in [0,1], that starts and ends at given points $\mathbf{p}_0$ and $\mathbf{p}_1$, and whose *second derivative* at t = ½ is a given vector $\mathbf{v}$. Derive the constraint matrix for this curve (show your work). (5 pts)

(b) Given the basis matrix for a quadratic spline defined as follows:

$$B = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

What are the basis functions for this spline? (5 pts)

(c) Consider 3 ways of defining a spline based on *n* control points (where $n > 4$):

1. a degree-3 Catmull-Rom spline,
2. a degree-3 endpoint-interpolating B-spline, and
3. a degree-(n-1) Bezier curve

For each of the following properties, circle the splines that satisfy the property. (6 pts)

Interpolates all control points:

Catmull-Rom B-spline Bezier curve

Lie entirely in the convex hull of the control points

Catmull-Rom B-spline Bezier curve

Has local control

Catmull-Rom B-spline Bezier curve

is C^1 continuous

Catmull-Rom B-spline Bezier curve

is C^2 continuous

Catmull-Rom B-spline Bezier curve

is C^3 continuous

Catmull-Rom B-spline Bezier curve

3. Animation (14 points)

(a) For each of the following animation techniques, briefly describe the idea and explain one of its advantages and one of its disadvantages. Give an example of a type of motion that the technique would be good for, and another type of motion for which it would be unsuitable. (9 pts)

1. Keyframing

2. Physics-based simulation

3. Motion capture

(b) Name and briefly describe 5 animation principles (e.g. “squash and stretch”). (5 pts)

4. Ray tracing (14 points)

(a) Suppose we are using a rendering window n_x pixels wide and n_y pixels tall, and a view volume defined in eye coordinates as in problem 1. Tell how to map integer pixel coordinates (i,j) to eye coordinates so that the bottom left corner of the bottom left pixel (which is pixel (0,0)) maps to the point (l,b,n), and the top right corner of the top right pixel (which is pixel $(n_x-1,\ n_y-1)$) maps to (r,t,n). Assume that pixel location (i,j) corresponds to the *center* of the pixel. (Your transformation should include a shift to account for this.) (5 pts)

(b) Let an infinite cylinder be defined as those points (x,y,z) that satisfy $x^2+y^2 = 1$. Given a ray defined by $\mathbf{r}(t) = \mathbf{p} + t\ \mathbf{v}$, where $\mathbf{p}$ is a given point and $\mathbf{v}$ is a vector, show how to compute the t-values of the intersection point(s) of the ray with the cylinder. (Derive a formula for t). (5 pts)

(c) Continuing with the problem in (b), let $\mathbf{p} = (-1, -1, 5)$ and $\mathbf{v} = (2, 1, 10)$. (4 pts)

Does the ray intersect the cylinder?

If so, what are the *t*-values and intersection points?

5. Radiosity (16 points)

(a) Suppose a surface has uniform emission E and reflection R (which tells the fraction of incoming radiance that is reflected by the surface). Suppose the surface forms a closed environment, which means it can be modeled as a single patch for purposes of computing radiosity. Also suppose that, to account for atmospheric effects, the form factor relating this patch with itself is ½, meaning that only half the radiosity that leaves the patch arrives back at the patch. Write an equation expressing the radiosity B of this environment in terms of E, R, and B. (4 pts)

(b) Suppose $R = 2/3$. What value of E is needed to achieve a radiosity of 9 units? (2 pts)

(c) For a scene with n patches, what is the size of the system of linear equations that must be solved to compute the radiosity? (2 pts)

(d) For a scene with *n* patches, how many form factors must be computed? (2 pts)

(e) For each of radiosity and ray tracing, name 3 limitations of the method and explain the reason for the limitation (e.g., it doesn't achieve effect X because of assumption Y). (6 pts)