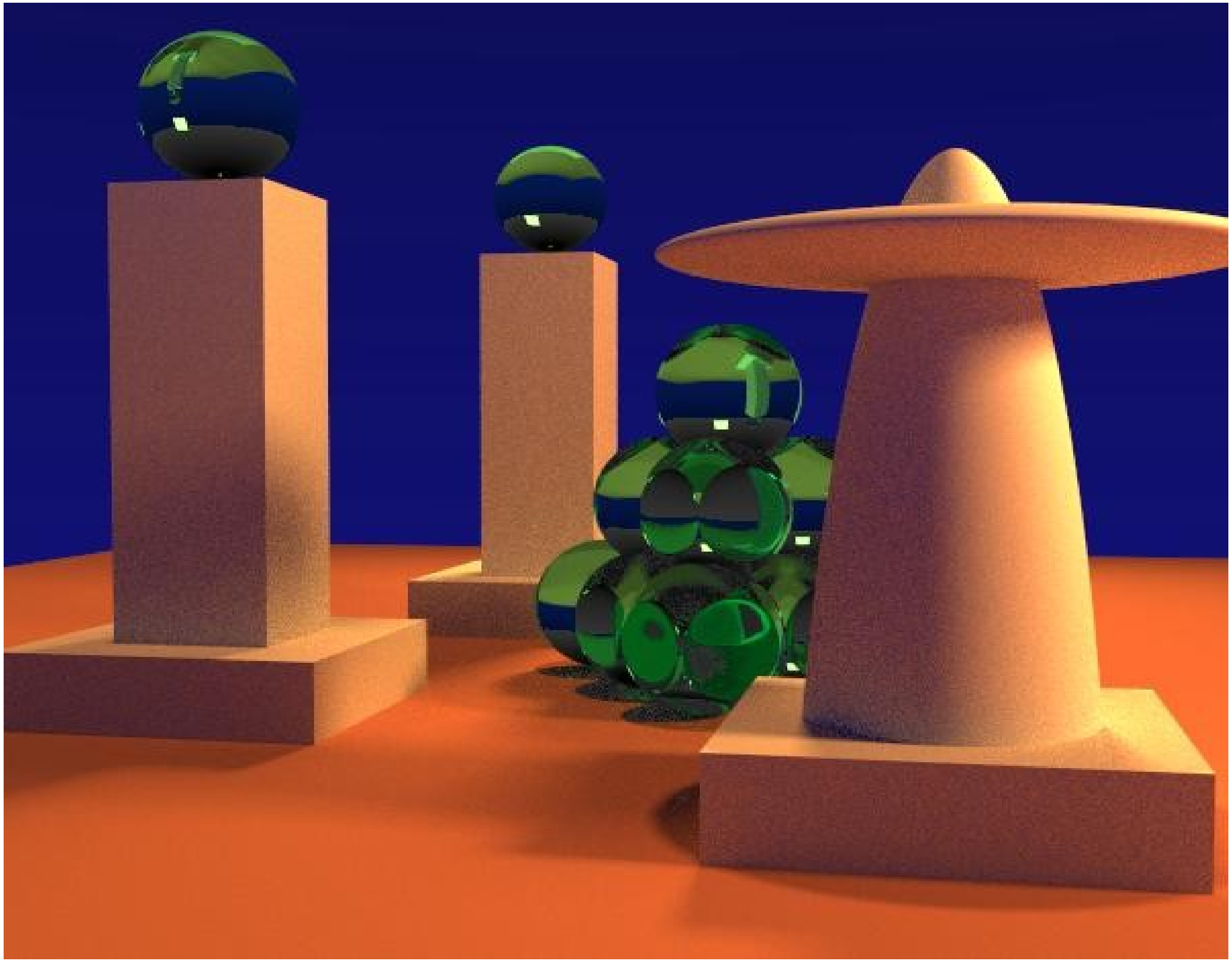
Monte Carlo Ray Tracing

EECS 487

November 22, 2006



outline

- rendering algorithms:
  - scan conversion
  - ray casting
  - ray tracing
  - monte carlo ray tracing

scan conversion

```
for each triangle T
  for each pixel in T
     color the pixel (if depth test ok)
```

scan conversion: analysis

- rendering window has p pixels (e.g. ~1 million)
- scene has n triangles (e.g. ~200,000)
- average depth complexity is d (e.g. d < 4)
- what is an upper bound on # steps to render?

scan conversion: analysis

number of steps = n + p*d

e.g.: 4.2 million

ray casting

```
for each pixel
   construct corresponding ray r
   intersect r with scene
   compute color via lighting, textures
```

ray casting: analysis

- rendering window has p pixels (e.g. ~1 million)
- scene has n triangles (e.g. ~200,000)
- (depth complexity not relevant)
- what is an upper bound on # steps to render? (assuming ray intersections are brute force)

ray casting: analysis

number of steps = p*n

e.g. 200 billion

(50,000 times slower than scan conversion)

ray tracing

```
for each pixel
   construct corresponding ray r
   intersect r with scene
   compute color via lighting, textures
   spawn 2 more rays and recurse
```

ray tracing: analysis

- rendering window has p pixels (e.g. ~1 million)
- scene has n triangles (e.g. ~200,000)
- depth of recursion is r (e.g. 5)
- what is an upper bound on # steps to render? (assuming ray intersections are brute force)

ray tracing: analysis

number of ray tests depends on level of recursion:

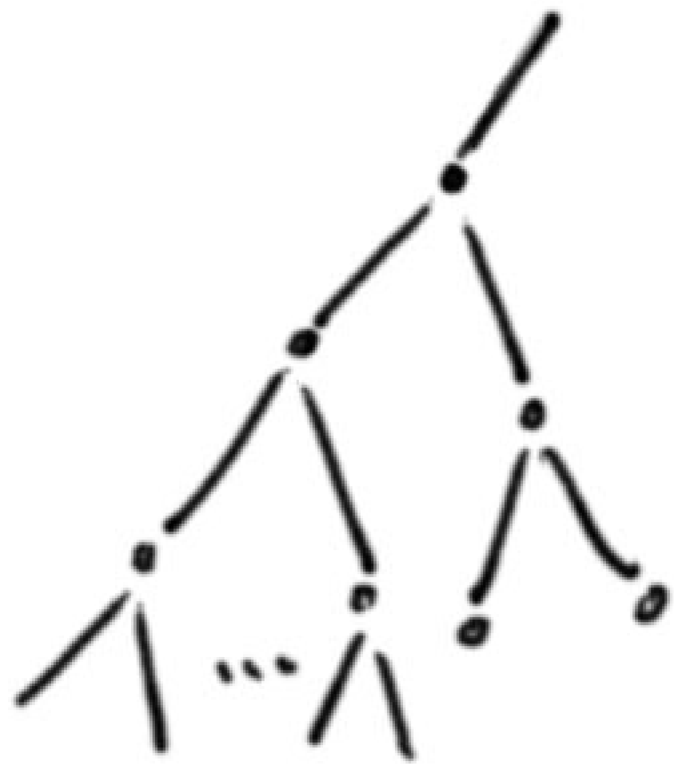
1 level: 1 test (per pixel)

2 levels: 3 tests

3 levels: 7 tests

r levels: $2^r - 1$ tests

round up: $\sim 2^r$ tests



ray tracing: analysis

ray tests (per pixel): 2^r tests

steps per ray test: n

total number of steps: $p * n * 2^r$

e.g.: 32 times slower than ray casting
in our example (ignoring shadow rays)

ray tracing analysis

- not so bad!
- can we make it slower?

ray tracing analysis

- not so bad!

- can we make it slower?
- yes!

monte carlo ray tracing

```
for each pixel
   construct corresponding ray r
   intersect r with scene
   compute color via lighting, textures
   spawn multiple additional rays
     and recurse
```

Q: how is this different from ray tracing?

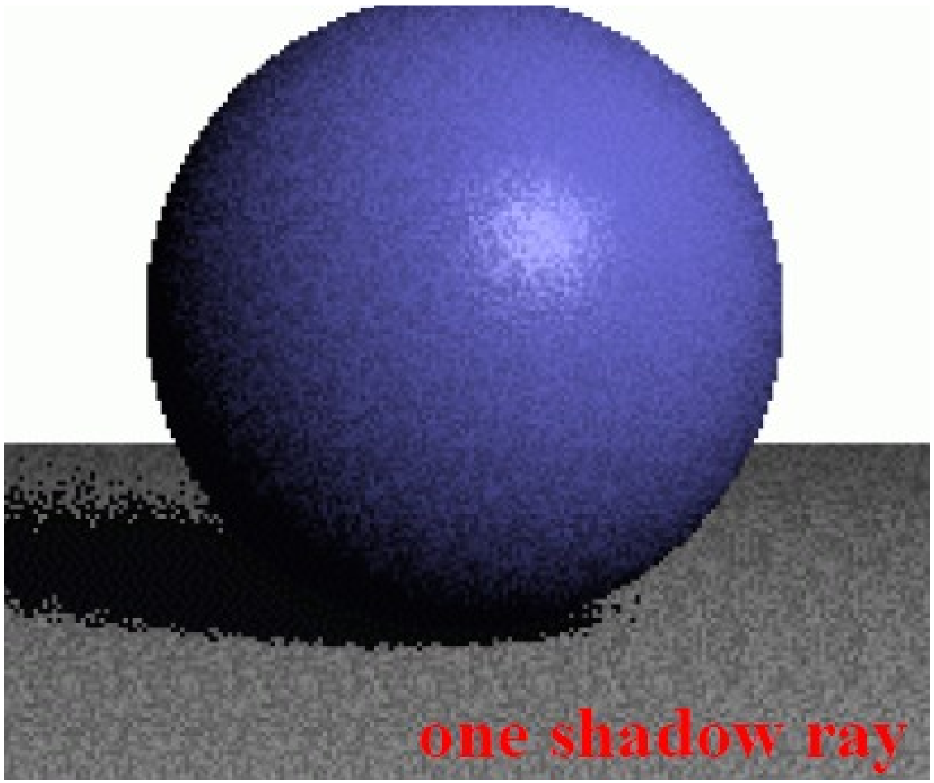
monte carlo ray tracing

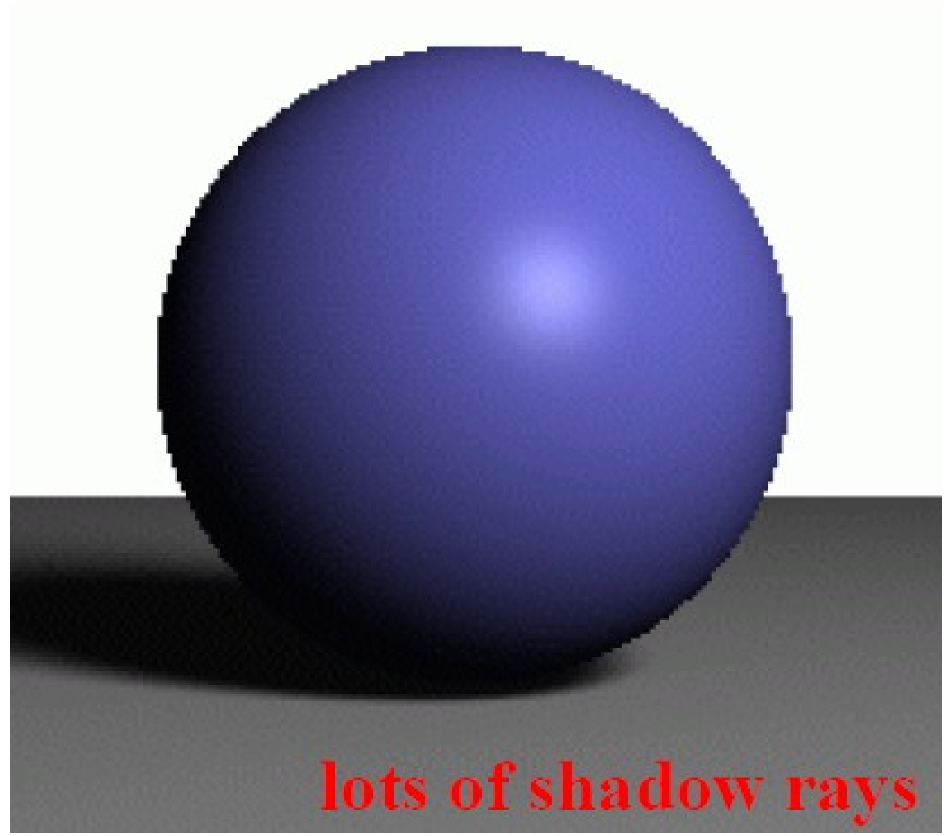
```
for each pixel
   construct corresponding ray r
   intersect r with scene
   compute color via lighting, textures
   spawn multiple additional rays
     and recurse
```

A: “multiple” instead of “2”

Q: why cast all these additional rays?

- A: get better simulation of global illumination
- e.g. soft shadows:
  - instead of 1 shadow ray to each point light,
  - cast multiple (random) rays to each *area* light
  - or: cast 1 (random) ray to each area light
  - fewer samples yields more “noise”





groups.csail.mit.edu/graphics/classes/6.837/F03/lectures/19_MonteCarlo.pdf

other effects

- soft shadows
- ?

other effects

- soft shadows
- glossy reflection
- color bleeding
- motion blur
- depth of field
- caustics?

monte carlo ray tracing: analysis

same as ray tracing, except the “branching factor” of the ray tree is not 2

call it b (e.g. b = 100)

recursion level: r (e.g. r = 5)

ray tests (per pixel): b^r tests

monte carlo ray tracing: analysis

total number of steps: $p * n * b^r$

$(b/2)^r$ times more work

(e.g. 50^5, or 300 million times more work than plain ray-tracing in our example)

observations

- actually, maybe b = 100 was a tad high... (but low values produce noise)
- brute force ray tests are a bad idea here (smarter method could be much faster)
- need to limit the depth of recursion (recurse when it will matter)
- and the number of rays cast
- should avoid work that makes no contribution

modification: monte carlo path tracing

- trace only 1 secondary ray per recursion
- but trace many primary rays per pixel
- (performs antialiasing as well)

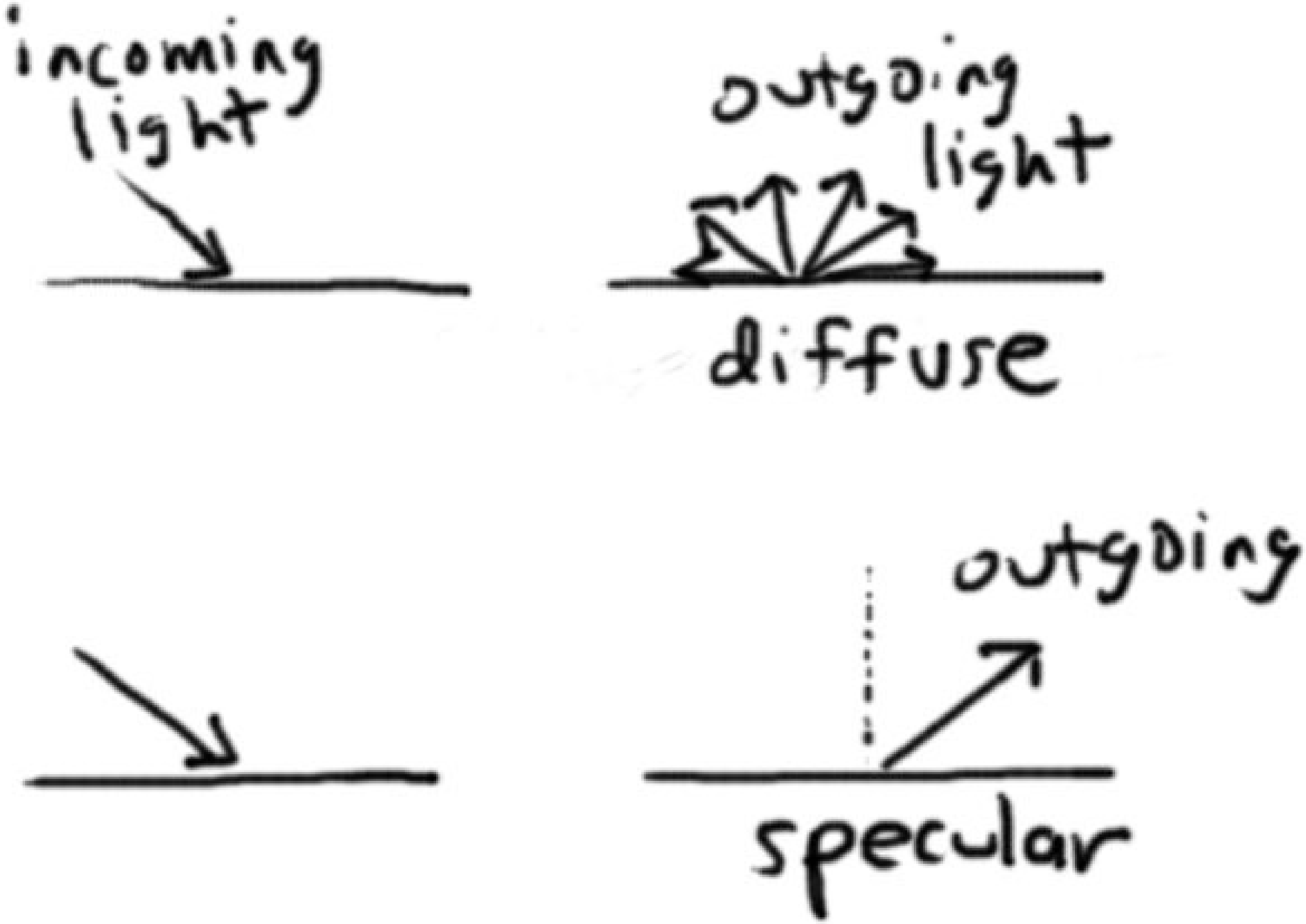
monte carlo path tracing

```
trace ray:
  find ray intersection with nearest object
  shade object


shade object:
  sample incoming light(via 1 random ray)
  shade using BRDF
```

Digression: what is a “BRDF”?

Simple BRDFs: diffuse, specular reflection



General BRDFs

- Most real materials do not correspond to either of those extremes (diffuse or specular)
- E.g., a glossy surface:

General BRDFs

- BRDF:
  bi-directional reflectance distribution function
- For each incoming direction,
  tells how much light will be reflected
  in each outgoing direction

OK but, why “monte carlo”??

next few slides sampled from:

groups.csail.mit.edu/graphics/classes/6.837/F03/lectures/19_MonteCarlo.pdf

and also:

http://www.cs.utah.edu/classes/cs6620/lecture-2006-03-24-6up.pdf

monte carlo integration

- want to evaluate: $$\int f(x)\,dx$$

- Use random variable x_i with uniform probability:

$$\frac{1}{n}\sum_{i=1}^{n} f(x_i)$$

improved version

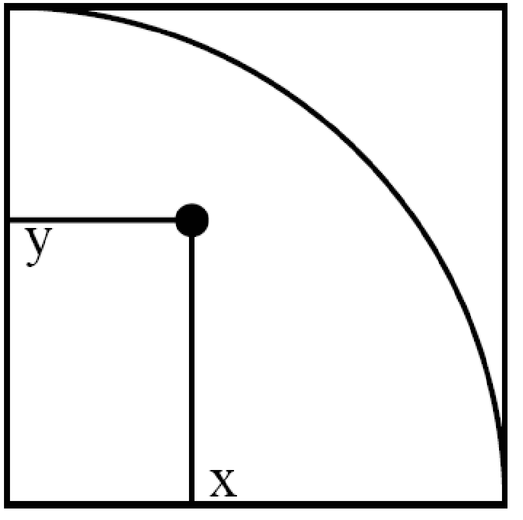
- Use random variable x_i *with probability p_i*

$$\frac{1}{n}\sum_{i=1}^{n}\frac{f(x_i)}{p_i}$$

- the whole trick is to choose the x_i and p_i

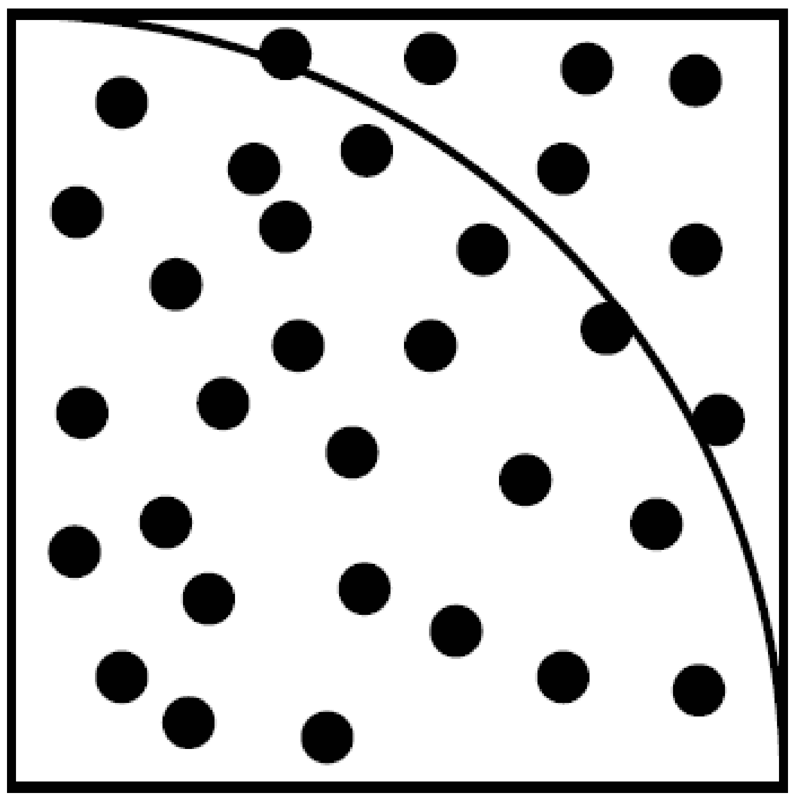
Example: monte carlo integration of π

- take a square
- take a random point (x,y) in the square
- test if it is in the ¼ circle ($x^2 + y^2 < 1$)
- run a lot of trials to estimate the probability
- the probability is $\pi/4$
- i.e.: your estimate times 4 is approximately π



Example: monte carlo integration of π

- error depends on number of trials



link to ray tracing

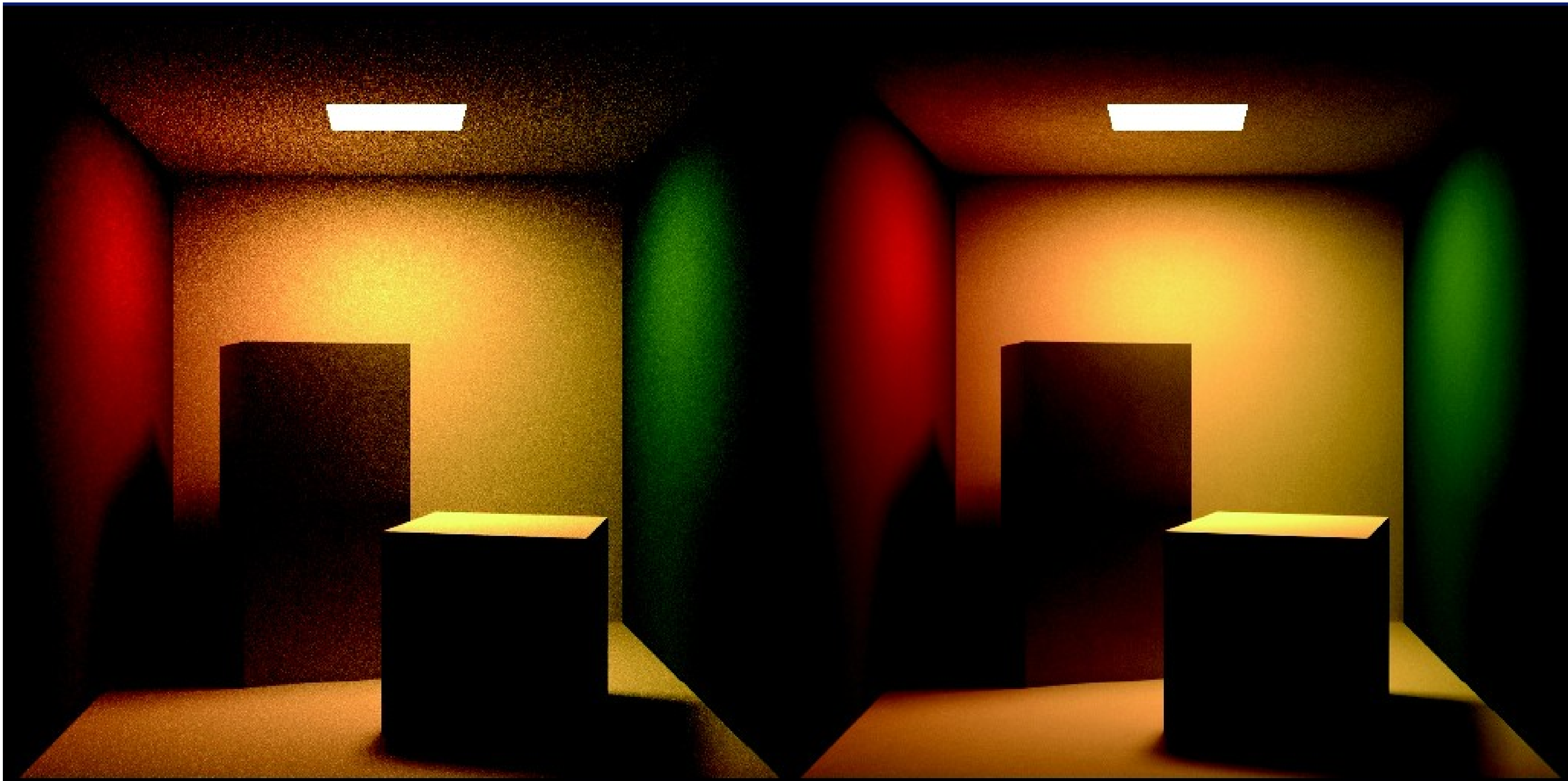
- Integration over light source area:
  - Soft shadows
- Integration over reflection angle:
  - Blurry reflections (gloss)
- Integration over transmitted angle:
  - Translucency (fuzzy transparency)

link to ray tracing

- Integration over camera lens:
  - Depth of field
- Integration over time:
  - Motion blur

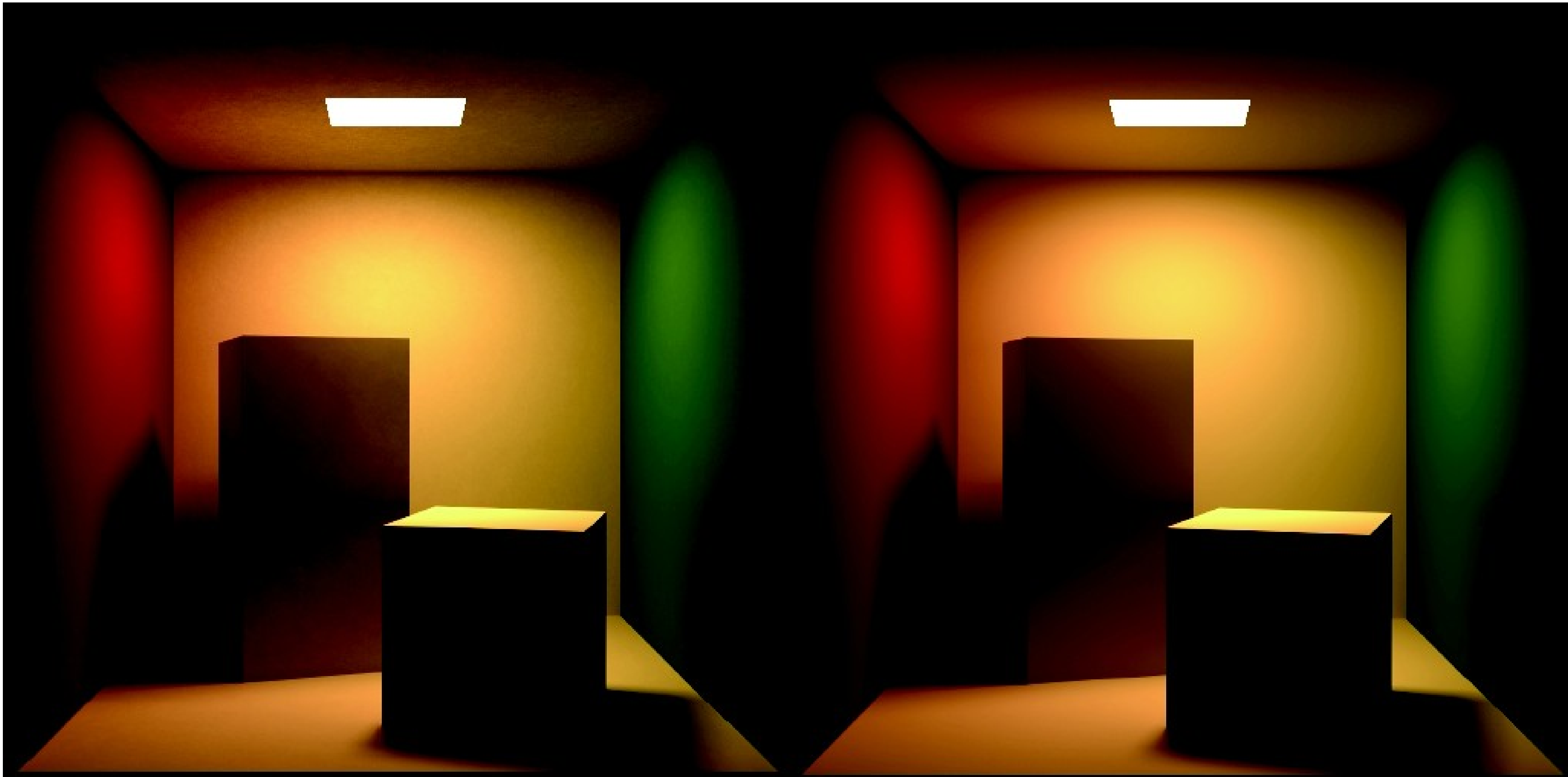
sampling strategies

- Pure Monte Carlo approach says to pick a random direction at each point
- Most rays will not hit a light source
- Kajiya style path tracing: pick a random light source and sample it randomly
    - Good convergence for scenes dominated by direct light



49 Samples per pixel

625 Samples per pixel



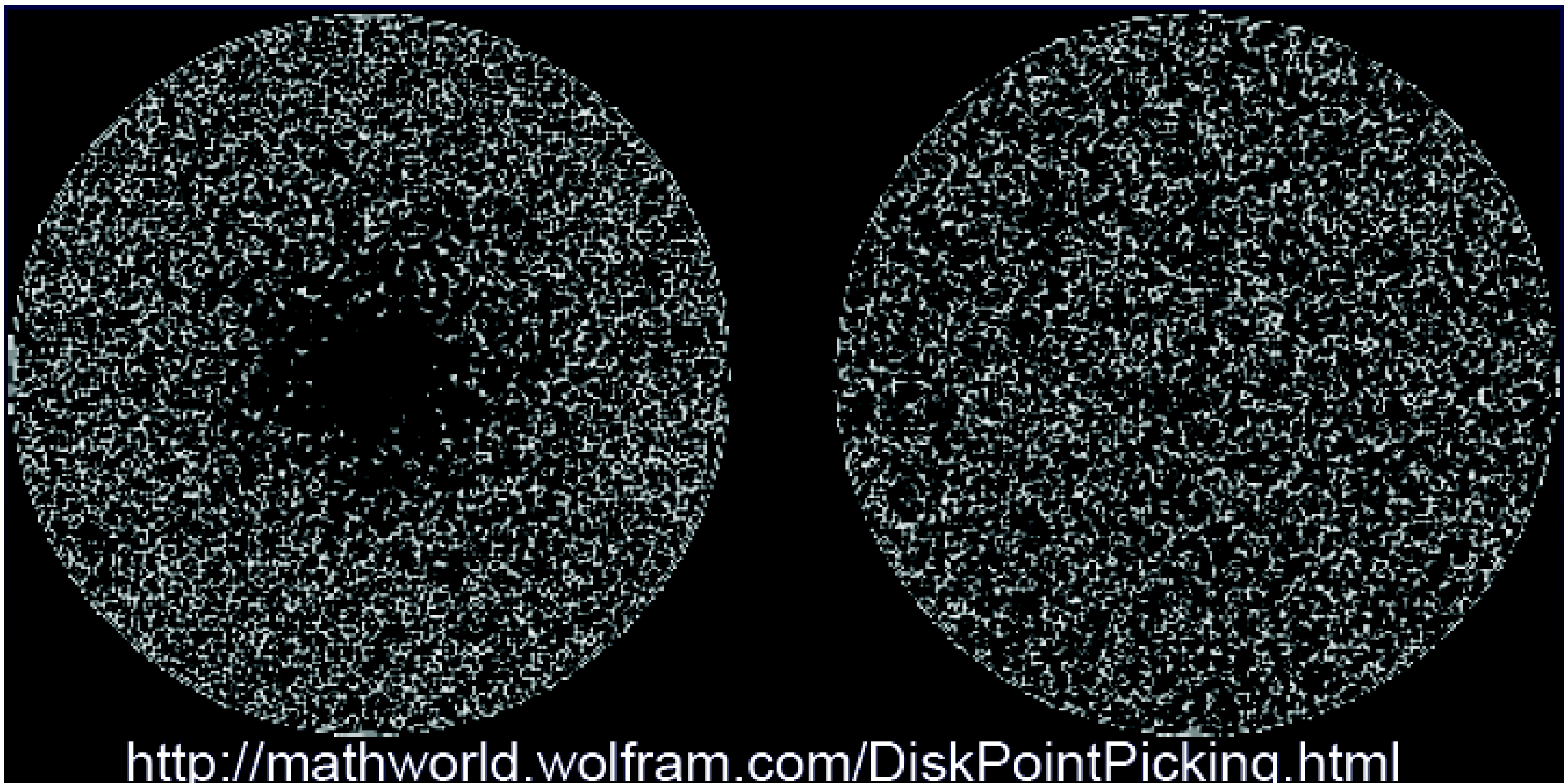
10000 Samples per pixel

100000 Samples per pixel

random sampling can be tricky

How to sample points on a disk uniformly?

wrong: right:



sampling a disk uniformly

- wrong:
  choose angle and radius uniformly:
  $\theta \in [0, 2\pi]$
  $r \in [0,1]$
  $x = r\cos(\theta)$, $y = r\sin(\theta)$

- Q: what's wrong with this?

sampling a disk uniformly

- wrong:
  choose angle and radius uniformly:
  $\theta \in [0, 2\pi]$
  $r \in [0,1]$
  $x = r\cos(\theta)$, $y = r\sin(\theta)$

- Q: what's wrong with this?
  A: samples are more crowded near center

sampling a disk uniformly

Right:

choose angle and r^2 uniformly:

$\theta \in [0, 2\pi]$

$r^2 \in [0,1]$ **note:** r^2, not r

$x = r\cos(\theta)$, $y = r\sin(\theta)$

Creates more samples at larger ~~radiuses~~ radii

monte carlo recap

- Turn integral into finite sum
- Use random samples
  - more samples = more accuracy (less noise)
- Very flexible
- Tweak sampling/probabilities for optimal result
- A lot of integration and probability theory
  to get things right

wrap up...

- project 4 due in 1 week
- project 5 out then (11/29)
- Thanksgiving break starts now...
  (well, after office hours)