Matrix Transformations

Lecture 3

Basic Graphics Transforms
- Translation
- Scaling
- Rotation
- Reflection
- Shear

All Can be Expressed As Linear Functions of the Original Coordinates :

$$x' = Ax + By + C$$
$$y' = Dx + Ey + F$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A & B & C \\ D & E & F \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

1

Affine Transformations

Lecture 3

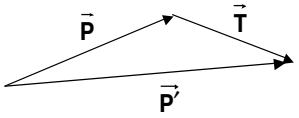
- Preserve Parallel Lines
- Finite Points Map to Finite Points
- Translation, Rotation, and Reflection Preserve :
  - Angles
  - Lengths

2

2-D Transformations - Translation

Lecture 3

$$x' = x + T_x$$
$$y' = y + T_y$$



Vectors : $\vec{P'} = \vec{P} + \vec{T}$

Matrices : $[P'] = [P] + [T]$

$$\begin{bmatrix} P'_x \\ P'_y \end{bmatrix} = \begin{bmatrix} P_x \\ P_y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix} = \begin{bmatrix} P_x + T_x \\ P_y + T_y \end{bmatrix}$$

Transform Polygons by Transforming Each Vertex

3

2-D Transformations - Scaling

Lecture 3

$$x' = S_x \cdot x$$
$$y' = S_y \cdot y$$

$$[P'] = [S][P]$$

$$\begin{bmatrix} P'_x \\ P'_y \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} P_x \\ P_y \end{bmatrix} = \begin{bmatrix} S_x \cdot P_x \\ S_y \cdot P_y \end{bmatrix}$$

Uniform Scaling : $S_x = S_y$

Differential Scaling : $S_x \neq S_y$

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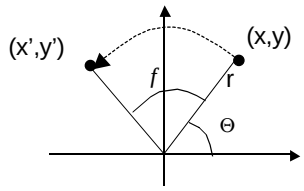
2-D Transformations - Rotation

Lecture 3



$$r\cos\Theta = x$$
$$r\sin\Theta = y$$

$$x' = r\cos(\Theta + f) = r\cos\Theta\cos f - r\sin\Theta\sin f$$
$$y' = r\sin(\Theta + f) = r\cos\Theta\sin f + r\sin\Theta\cos f$$

Rotation about the Origin

$$x' = x\cos f - y\sin f$$
$$y' = x\sin f + y\cos f$$

$$[\mathbf{P'}] = [\mathbf{R}(f)][\mathbf{P}]$$

$$\begin{bmatrix} P'_x \\ P'_y \end{bmatrix} = \begin{bmatrix} \cos f & -\sin f \\ \sin f & \cos f \end{bmatrix} \begin{bmatrix} P_x \\ P_y \end{bmatrix}$$

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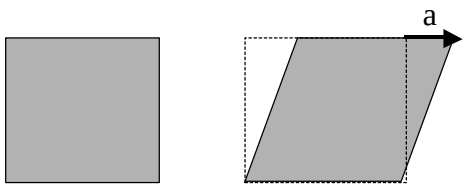
Shear Transforms

Lecture 3

Shear: An action or stress resulting from applied forces that causes or tends to causes two contiguous parts of a body to slide relatively to each other.



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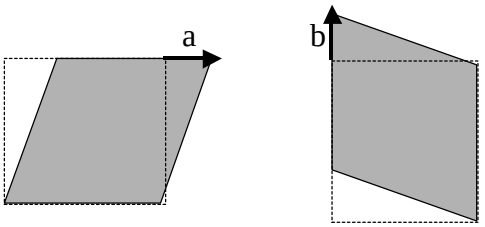
Shear Transforms

Lecture 3

$$\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}$$

Shear Translation in X Shear Translation in Y



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Shear Transforms

Lecture 3

Properties:

- It is an Affine Transform
- a and b are the proportionality constant

A Shear Transforms Applications:

- Fast and Efficient Rotation
- It will be used in Perspective Transformations

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Shear Transforms

Lecture 3

Shear Raster Rotation

$$\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}\begin{bmatrix} 1 & l \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q & -\sin q \\ \sin q & \cos q \end{bmatrix}$$

$$a = -\tan\left(\frac{q}{2}\right)$$

$$b = \sin q$$

$$l = -\tan\left(\frac{q}{2}\right)$$

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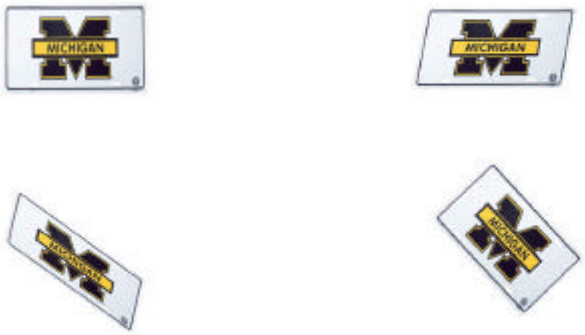
Shear Raster Rotation - Example

Lecture 3



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Reflection Transforms

Lecture 3

Reflection is a transform that allows vectors to be flipped about an axis as if reflected in a mirror.

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Reflection About YZ | Reflection About XZ | Reflection About XY

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2-D Transformations

Lecture 3

- Unify Transformation Operations
  - All Transformations Represented as Matrix Products

$$[\mathbf{T}] \equiv \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \qquad [\mathbf{S}] \equiv \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad [\mathbf{R}(f)] \equiv \begin{bmatrix} \cos f & -\sin f & 0 \\ \sin f & \cos f & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[\mathbf{P}'] = [\mathbf{T}][\mathbf{P}] \qquad [\mathbf{P}'] = [\mathbf{S}][\mathbf{P}] \qquad [\mathbf{P}'] = [\mathbf{R}(f)][\mathbf{P}]$$

Homogeneous Coordinates

$$[\mathbf{P}] = \begin{bmatrix} P_x \\ P_y \\ 1 \end{bmatrix}$$

NOT the z coordinate

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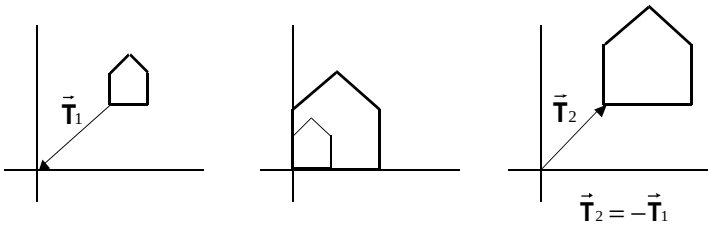
Transform Combinations

Lecture 3

- Scaling About a Point
  - Translate to Origin
  - Apply Scale Matrix
  - Translate Back to Original Position

$$[P'] = [T_2][S][T_1][P]$$

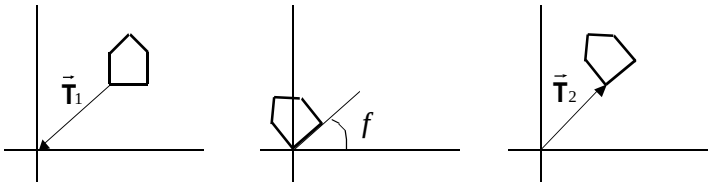


Transform Combinations

Lecture 3

- Rotation About a Point
  - Translate to Origin
  - Apply Rotation Matrix
  - Translate Back to Original Position

$$[P'] = [T_2][R(f)][T_1][P]$$



Interactive Computer Graphics

Lecture 3

Aliasing and Anti-Aliasing

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Aliasing and Anti-Aliasing

Lecture 3

We have a problem with the images we are creating. They have jagged edges or stair steps.

These effect is call Aliasing. It results from the process of converting an analog signal, the line, into a digital signal.

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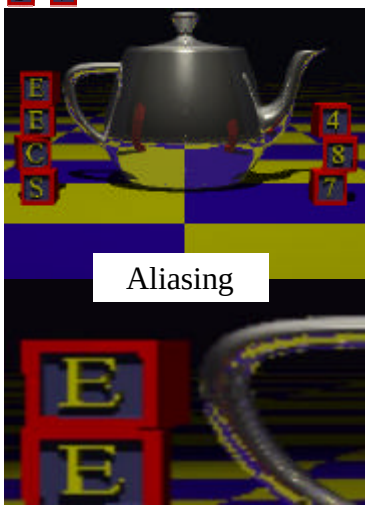
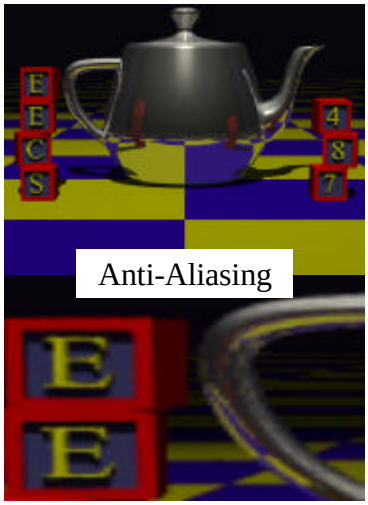
Aliasing and Anti-Aliasing -Examples

Lecture 3





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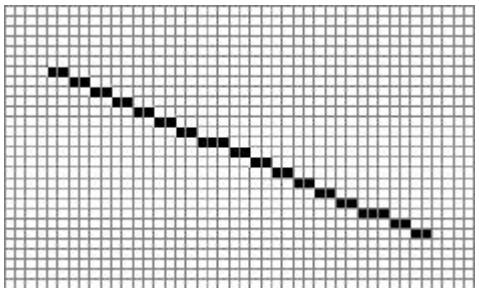
Aliasing and Anti-Aliasing

Lecture 3

To fully understand the causes of aliasing requires the use of Signal Processing and Fourier Transforms.

We can look at Aliasing from a more intuitive bases.



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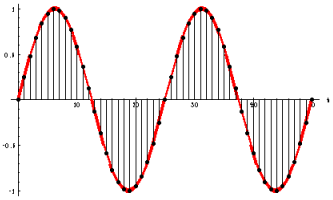
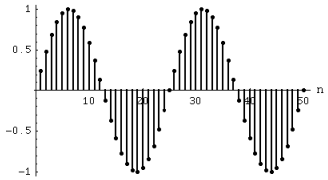
Aliasing and Anti-Aliasing

Lecture 3

Effects of Sampling





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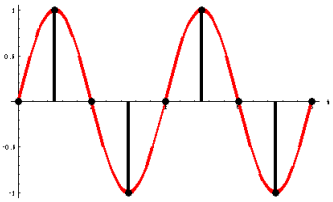
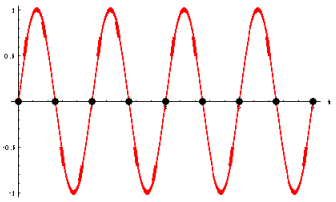
Aliasing and Anti-Aliasing

Lecture 3

Effects of Sampling





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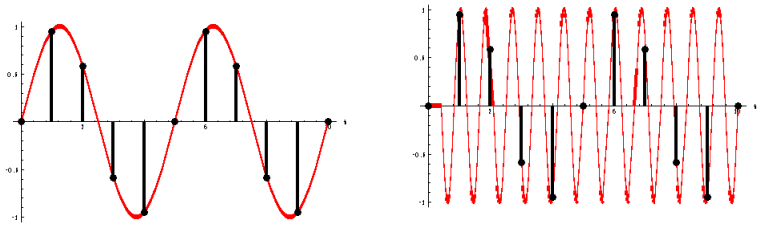
Aliasing and Anti-Aliasing

Lecture 3

Aliasing

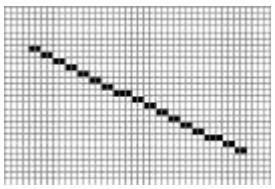


Aliasing and Anti-Aliasing

Lecture 3

Anti-Aliasing is the process that attempts to prevent or fix the problem.

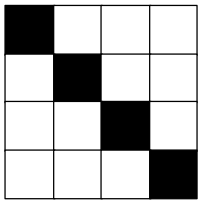
More Resolution
Unweighted Area Sampling
Weighted Area Sampling
Post image creation filtering

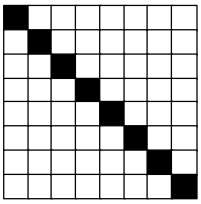


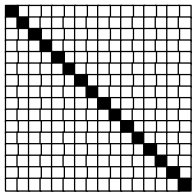
Aliasing and Anti-Aliasing

Lecture 3

More Resolution







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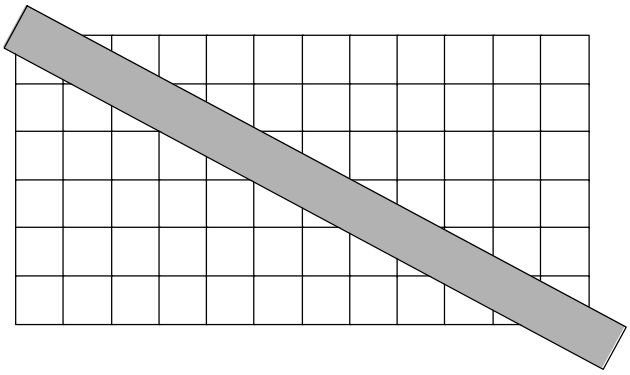
Aliasing and Anti-Aliasing

Lecture 3

Unweighted Area Sampling



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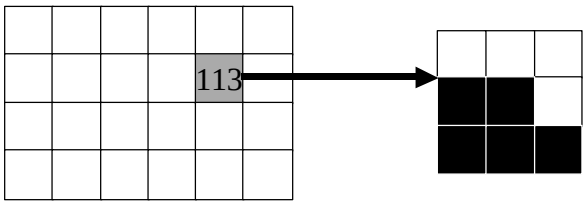
Aliasing and Anti-Aliasing

Lecture 3

Unweighted Area Sampling



Pixel

Sub Pixels

4/9

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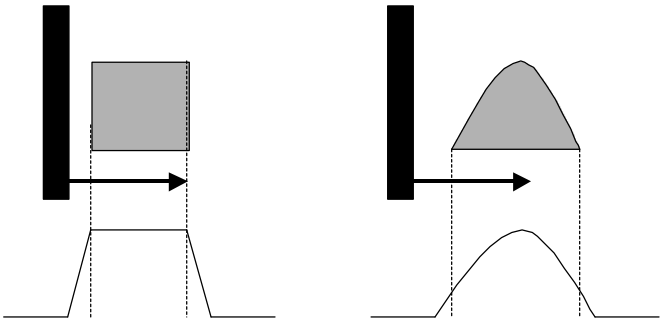
Aliasing and Anti-Aliasing

Lecture 3

Weighted Area Sampling



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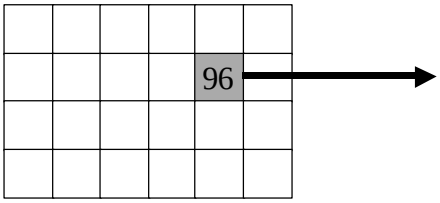
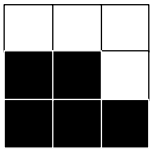
Aliasing and Anti-Aliasing

Lecture 3

Weighted Area Sampling



Pixel



Sub Pixels

1	2	1
2	4	2
1	2	1

Sub Pixels Weights

(1+2+1+2)/16

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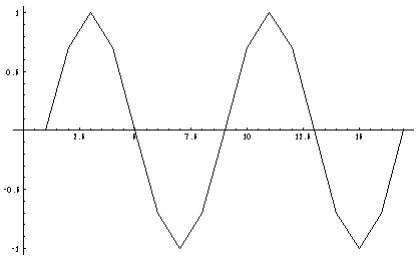
Aliasing and Anti-Aliasing

Lecture 3

Weighted Area Sampling



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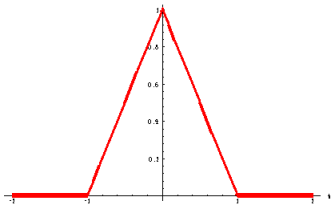
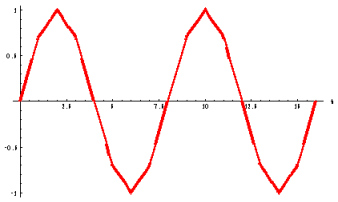
Aliasing and Anti-Aliasing

Lecture 3

Weighted Area Sampling





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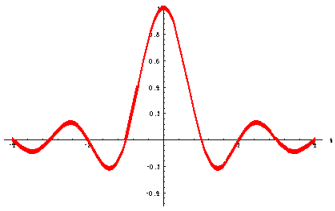
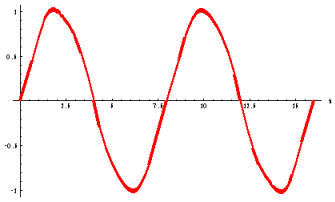
Aliasing and Anti-Aliasing

Lecture 3

Weighted Area Sampling





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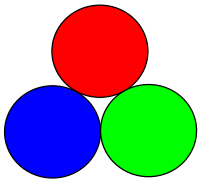
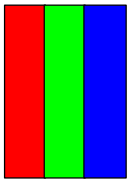
Aliasing and Anti-Aliasing

Lecture 3

Weighted Area Sampling



Video Pixel



LCD Pixel

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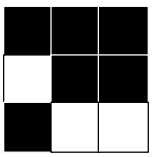
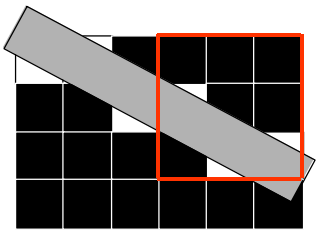
Aliasing and Anti-Aliasing

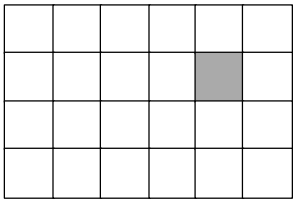
Lecture 3

Post image creation filtering



= 3/9 * 255 = 85





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Aliasing and Anti-Aliasing

Lecture 3

More on the Evil Aliasing Empire Later

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