



Ray Tracing I

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History

Ray Tracing came from the Physics of lens making. The process was that of drawing lines or rays through a glass shape to determine its lens properties. It is also related to early perspective painting technique of Durer.

The ideas of using rays to make computers images was first tried in the early 60's, but with the computer power of that time was so much slower than other methods that it was considered not to worth the effort.

By the early 80' s the computer power had developed and ray tracing was given another try. These images showed promise and lead to the research that has made today's images possible.

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Ray Tracing

```
for all pixels (x,y) {  
  for all objects {  
    compare z  
  }  
}
```

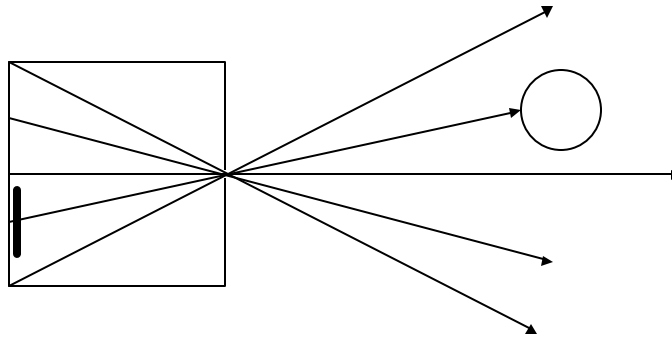
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Ray tracing uses geometric optics. We can look back at the pin hole camera to see the basics of ray tracing.



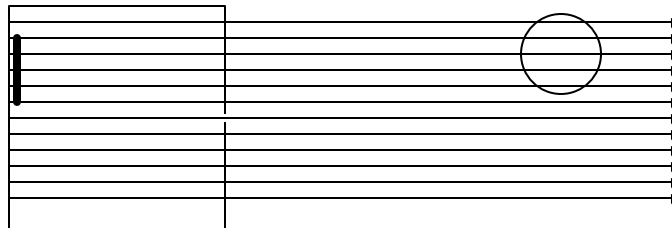
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Ray tracing can also be used to generate parallel view images. In this mode we look from each pixel in the same direction.



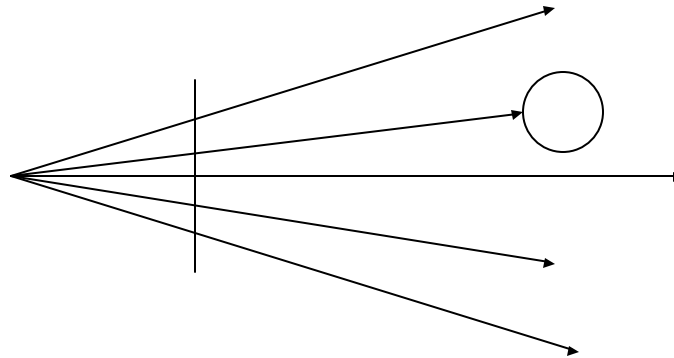
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Remember the Camera Models.



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The Camera Model is used to generate the initial rays. In ray tracing we do not use the Viewing Transformations.

The relationship between pixel and initial ray direction is simple for the orthographic camera model. Each ray takes on the X-Y-Z position of the pixel and all rays have the same vector direction, that of the look vector.

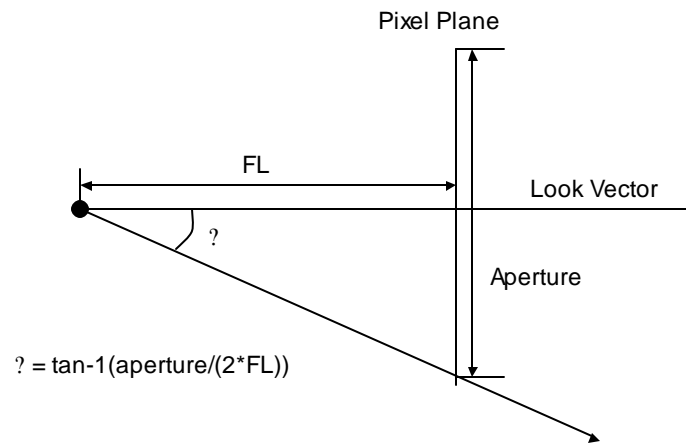
The relationship between pixel and initial ray direction is more complex for the Perspective camera model. It is a rectangular to polar transformation.

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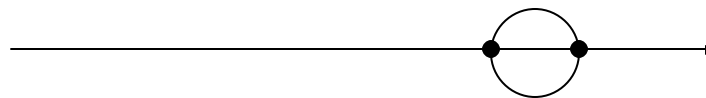
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The core of any ray tracing systems, as well as the bottle neck, is the finding of the intersection points between an object and a ray. All ray intersection problems boil down to the mathematical process of finding roots.



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Sphere Intersection

The vector is defined as:

$$\mathbf{P}_{\text{origin}} = \mathbf{P}_o = [X_o, Y_o, Z_o]$$

$$\mathbf{P}_{\text{direction}} = \mathbf{P}_d = [X_d, Y_d, Z_d] \text{ where } \mathbf{P}_d \text{ is normalized.}$$

This defines a ray as a set of points on the line

$$\mathbf{P}(t) = \mathbf{P}_o + \mathbf{P}_d * t, \text{ where } t > 0$$

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The sphere is defined as:

$$\text{Sphere center} = \mathbf{S}_c = [X_c, Y_c, Z_c]$$

$$\text{Sphere radius} = \mathbf{S}_r = r$$

The sphere's surface is defined as all points $[X_s, Y_s, Z_s]$ in the implicit equation:

$$(X_s - X_c)^2 + (Y_s - Y_c)^2 + (Z_s - Z_c)^2 - S_r^2 = 0$$

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The intersection of the sphere and the ray can now be found by placing the ray equation into the sphere equations.

$$\begin{aligned}X &= X_0 + X_d * t \\Y &= Y_0 + Y_d * t \\Z &= Z_0 + Z_d * t\end{aligned}$$

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Resulting in:

$$(X_0 + X_d * t - X_c)^2 + (Y_0 + Y_d * t - Y_c)^2 + (Z_0 + Z_d * t - Z_c)^2 - S_r^2 = 0$$

Simplified this results in:

$$\begin{aligned}&(X_d^2 + Y_d^2 + Z_d^2)t^2 \\&+ 2*(X_d*(X_0 - X_c) + Y_d*(Y_0 - Y_c) + Z_d*(Z_0 - Z_c))*t \\&+ (X_0 - X_c)^2 + (Y_0 - Y_c)^2 + (Z_0 - Z_c)^2 - S_r^2 = 0\end{aligned}$$

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We now solve for t . Note that the above equation is a simple quadratic

$$\begin{aligned} & A \cdot t^2 \quad \text{where } A = (X_d^2 + Y_d^2 + Z_d^2) \\ & + B \cdot t \quad \text{where } B = 2 \cdot (X_d \cdot (X_0 - X_c) + Y_d \cdot (Y_0 - Y_c) + Z_d \cdot (Z_0 - Z_c)) \\ & + C = 0 \quad \text{where } C = (X_0 - X_c)^2 + (Y_0 - Y_c)^2 + (Z_0 - Z_c)^2 - S_r^2 \end{aligned}$$

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Because the direction ray is normalized A is always equal to 1. So the solutions for t can be found by using the quadratic formula:

$$t_0 = (-B - \sqrt{B^2 - 4 \cdot C}) / 2$$

$$t_1 = (-B + \sqrt{B^2 - 4 \cdot C}) / 2$$

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- Only the real roots show that the ray and sphere intersect.
- Non-real roots indicate the sphere was missed. Negative values of t are not used and indicate that the ray started in the sphere and only the positive roots are valid.
- We can then compare t_0 and t_1 to find out which is smaller indicating the point on the sphere that is closer to the ray starting point \mathbf{P}_o .

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Once we have selected t_0 or t_1 we can then place it back in the ray definition and find the true intersection point in X, Y, Z .

$$\mathbf{P}(\text{intersection}) = \mathbf{P}_o + \mathbf{P}_d * t_{0/1}$$

We can then find the sphere's surface normal at this point and other properties needed to render the image.

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Plane Intersection

For the plane use the same definition of the ray. The definition of a plane is:

$$\text{Plane} = A*x + B*y + C*z + D = 0$$

where $A^2 + B^2 + C^2 = 1$

The unit normal of the Plane defines A, B, C:

$$P_{\text{normal}} = P_n = [A \ B \ C]$$

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D is defined as the distance from coordinate system origin [0 0 0]. The sign of D determines which side of the plane the system origin is located. Again we substitute the Ray into the plane definition.

$$A*(X_0 + X_d*t) + B*(Y_0 + Y_d*t) + C*(Z_0 + Z_d*t) + D = 0$$

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We again solve for t:

$$t = -(A * X_0 + B * Y_0 + C * Z_0 + D) / (A * X_d + B * Y_d + C * Z_d)$$

We can write this in vector notation:

$$t = -(\mathbf{P}_n \cdot \mathbf{P}_o + D) / (\mathbf{P}_n \cdot \mathbf{P}_d)$$

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Now we can look at the components of the solution for t and find that we can test for simple things to save time and computations. First let's look at the bottom part:

$$V_d = \mathbf{P}_n \cdot \mathbf{P}_d = (A * X_d + B * Y_d + C * Z_d)$$

If this is equal to zero then the ray is parallel to the plane and there is no intersection. Even if the ray lies in the plane you can not have an intersection with the edge of a plane.

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Once again we can put t back into the ray equation and find the intersection point.

$$P(\text{intersection}) = P_o + P_d * t$$

We can then quickly determine if this intersection point is within a four sided polygon by testing min and max X,Y,Z points. These polygons are often used to bound complex shapes and object. The ray tracer would first see if the ray hits the bounding volume before it proceeds to find intersection of the complex objects contained in the volume.

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Constructive Solid Geometry

- Find intersection along ray with each object that makes up the solid.
- Follow CSG tree to combine intersection pairs to find the final intersection pair.

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References

An Introduction to Ray Tracing

by Andrew S. Glassner (Editor)

Academic Press; ISBN: 0122861604

Ray Tracing Worlds With Pov-Ray

Book and 2 Disks

by Alexander Enzmann, Lutz Kretschmar, Chris Young

ISBN: 1878739646

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