Lecture 16

Fractals, mountains, and trees

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Fractals

Lecture 16

- Coastlines
- Snowflakes
- Sponges
- Mountains, terrains
- Trees, bushes

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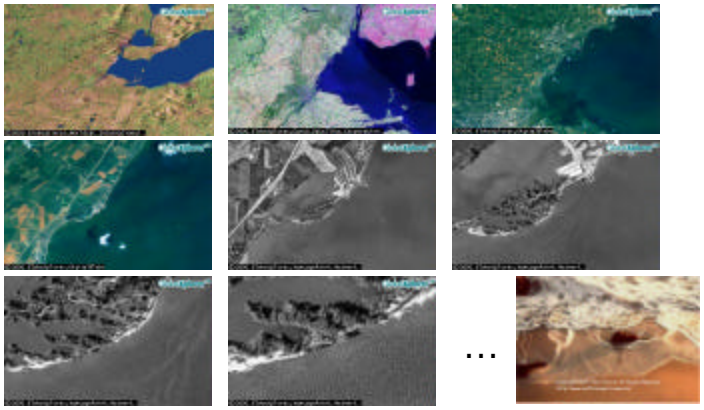
Coastline

Lecture 16

- Zooming in



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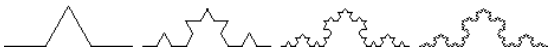
Coastline

Lecture 16

- How to represent and store all that?
  - Cubic Bezier curves?
- How do we generate it on the fly maybe?
- Koch curve 
  - What is the length of this curve?
  - 4*(1/3), 16*(1/9), 64*(1/27), … ?

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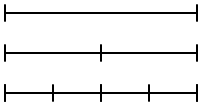
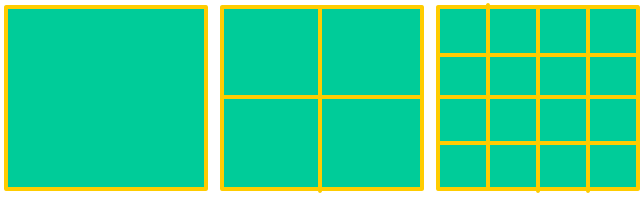
Fractal dimension

Lecture 16

- Let's go back to simple ones…
  - line: dimension = 1
    - 1, $2*(1/2)^1$,$4*(1/4)^1$, …
  - square: dimension = 2
    - 1, $4*(1/2)^2$, $16*(1/4)^2$, …





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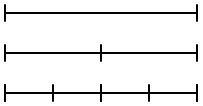
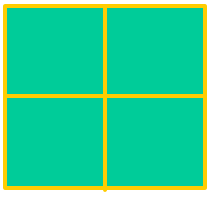
Fractal dimension

Lecture 16

- Line
  - scaling factor s=1/2
  - number of subparts n=2
  - d = log n / log (1/s) = 1
- Square
  - s = 1/2
  - n = 4
  - d = log 4 / log 2 = 2





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Fractal dimension

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- Dimension of Koch curve is the smallest d such that
  - $4*(1/3)^d$, $16*(1/9)^d$, $64*(1/27)^d$, … is not infinity
  - that is $(1/3^k)^d = 1/4^k$, so that $d * \log 3 = \log 4$
  - and $d = \log 4 / \log 3$? 1.26
    - n = 4, s = 1/3

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L-systems

Lecture 16

- How can we produce such objects?
- L-system is
  - symbols
    - language describing a 2d/3d scene
  - an axiom
    - starting point
  - rewriting rules

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L-systems

Lecture 16

- Lindenmayer 1968
- Turtle graphics
  - F draw forward
  - f move forward
  - + turn left
  - - turn right
  - [push current state onto stack
  -] pop current state from the stack

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Koch l-system

Lecture 16

- F+F--F+F
  angle=(2?)/6

```
Koch {
  Angle 6
  Axiom F
  F=F+F--F+F
}
```

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Generating Koch' s snowflake

Lecture 16

- Start: F
- Generation 1:
  - F+F--F+F

```
Koch {
  Angle 6
  Axiom F
  F=F+F--F+F
}
```

- Generation 2:
  - F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F

trees in 2d

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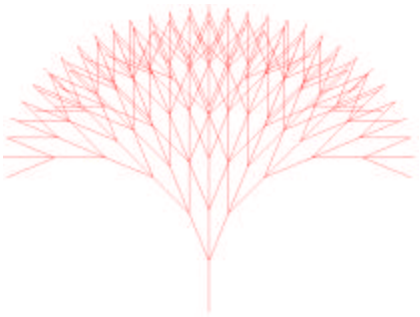
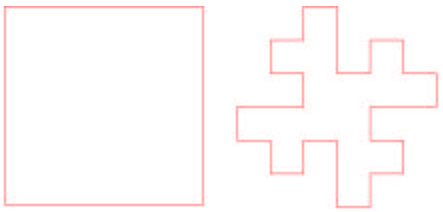
- Tree

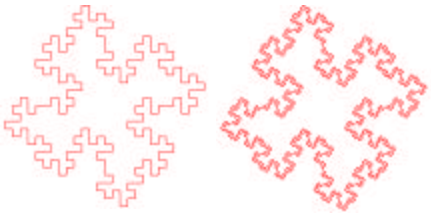


```
Tree {
  Angle 16
  Axiom ++++FS
  S=+[FS]-[FS]-[FS]
}
```

- Koch Island



```
KochIsland {
  Angle 4
  Axiom F+F+F+F
  F=F+F-F-FF+F+F-F
}
```



- Similarly:
  - 3d transforms
  - rotations
  - nested transforms
  - colors
  - position

- Przemyslaw Prusinkiewicz, U. of Calgary

Terrain modeling

Lecture 16

- Fractal mountains
  - geometry
  - colors
  - vegetation

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Creating fractal mountains

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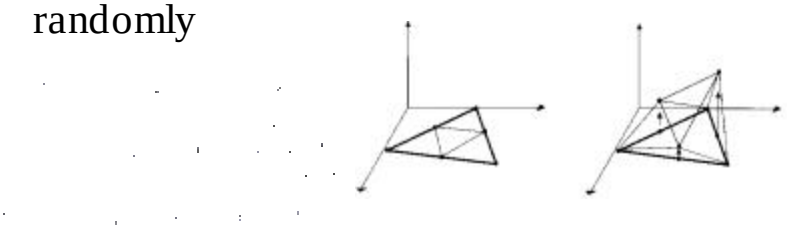
- Start with planar triangulation
- Let user displace coarse triangles
- Recursively subdivide and displace randomly



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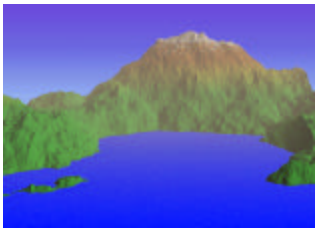
Elevation

Lecture 16

- Elevation governs:
  - color
    - snow
    - grass
  - trees distribution



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Sky, clouds

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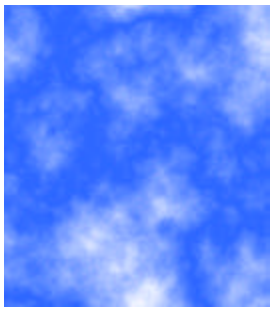
- Cloud texture

$$I(x,y) = \sum_{i=1}^{n} c_i \sin(k_i^x x + p_i^x) \sum_{i=1}^{n} c_i \sin(k_i^y y + p_i^y)$$

$k_{i+1}^x = 2k_i^x$

$k_{i+1}^y = 2k_i^y$

$c_{i+1} = 0.707 c_i$



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Smooth fractals

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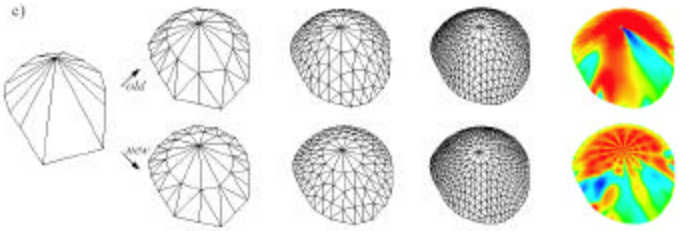
- Subdivision surfaces
  - generated from simple shapes using simple rules
  - curvature is fractal



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