Chapter 7, Problem 6.

a) Denote $d^{(n)}$ for the estimate after the *n*th sample.

$$d^{(1)} = r_4 - t_4$$

$$d^{(2)} = u(r_3 - t_3) + (1-u)(r_4 - t_4)$$

$$d^{(3)} = u(r_2 - t_2) + (1-u)\left[u(r_3 - t_3) + (1-u)(r_4 - t_4)\right]$$

$$= u(r_2 - t_2) + (1-u)u(r_3 - t_3) + (1-u)^2(r_4 - t_4)$$

$$d^{(4)} = u(r_1 - t_1) + (1-u)d^{(3)}$$

$$= u(r_1 - t_1) + (1-u)u(r_2 - t_2) + (1-u)^2 u(r_3 - t_3) + (1-u)^3(r_4 - t_4)$$

b)

$$d^{(n)} = u\sum_{j=1}^{n-1}(1-u)^{j-1}(r_j - t_j) + (1-u)^{n-1}(r_n - t_n)$$

Given that u is <0, as $u \to \infty$**,** $(1-u)^{\infty} \to 0$**,** thus the last term becomes zero, we have the following generalized formula:

$$d^{(\infty)} = \frac{u}{1-u}\sum_{j=1}^{\infty}(1-u)^{j}(r_j - t_j)$$