Recitation 6

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Topics: (Same as Chun-Hao’s handout)

Problems:

1) Let $X$ be a Poisson random variable, i.e. $p_X(x) = \frac{\lambda^k e^{-\lambda}}{k!}, k = 0,1,2,...$. Find $E[X]$, $E[X^2]$, $\text{var}[X]$, and $E[1/(X+1)]$. Also find $P[X \leq n]$.

2) In a certain discussion section of a certain sophomore-level EECS course, there were 20 students. Each student came to class with probability .9 independent of all other students.
   a. What is the probability that exactly 12 students were in class on a given day?
   b. Letting $X$ be the number of students in class on a given day, find the pmf of $X$.
   c. Find the expected value of $X$.
   d. Find the probability that exactly one student shows up to class. (This event did in fact occur on a cold November morning in 2001! Had the University not set rules against it, the student who showed up would have received the solutions to all remaining projects. Moral of the story: come to discussion!)

3) (Gubner, Problem 24, pg. 113) Let $X$ be a random variable with mean $m$ and variance $\sigma^2$. Find the constant $c$ that best approximates the random variable $X$ in the sense that $c$ minimizes the mean squared error $E[(X-c)^2]$.

4) (Signal processing/Communications) Let $X$ and $Y$ be two zero-mean random variables with correlation coefficient $\rho$. We wish to estimate $X$ based on $Y$. Find the minimum mean squared error (MMSE) estimate of $X$ based on observation $Y$. In other words, find the function $f(Y)$ such that $E[(X-f(Y))^2]$ is minimized. This is a difficult problem to solve in general, so, restrict the function $f(Y)$ to be linear, i.e., $f(Y) = aY$ where $a$ is a constant. (What if $X$ and $Y$ were not zero-mean?)

5) Gubner, Problem 30, pg. 114 (Betting on fair games)

6) Revisit an old problem:
   A casino has come up with a new game. It is a combination of a roulette wheel and a fair coin. The roulette wheel has the numbers 00, 0, and 1 through 36 (a total of 38 slots). Guests can bet on each of the 38 numbers (no splitting, red/black, or odd/even bets). Guests place their bets, and the wheel is spun. The ball will randomly land in one of the 38 slots. Then the coin is flipped. If the outcome of the coin toss is heads, winners are paid at a rate of 1:70. If it’s tails, winners are paid at the normal rate of 1:35.
   a) What is the probability of winning for the player?
   b) Is this a fair game to play for a player?