us subtract the mean and divide by \( \sqrt{N} \), the rate at which the distribution spreads. Accordingly, consider the random variable

\[
\bar{S} \triangleq \frac{S - \bar{S}}{\sqrt{N}} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} (X_i - \bar{X}),
\]

(3.1.12)

whose mean is 0 and whose variance is \( \sigma^2_X \). The asymptotic distribution of \( \bar{S} \) is given by the following well known law of large numbers:

Figure 3.1.2: The probability densities of: (a) \( S \), (b) \( \bar{S} \), and (c) \( \langle S \rangle - \bar{S} \), when the \( X_i \)'s are uniform on \([0,1]\).
Figure 3.1.3: The pmf’s of: (a) $S$, (b) $\bar{S}$, and (c) $\langle S \rangle - \bar{S}$, when the $X_i$’s are binary with $p(1) = .25$.

**Theorem 3.1.2 (The Central Limit Theorem)** Let $\{X_i\}$ be a real-valued, IID random process with finite mean and finite variance. Then the probability distribution of $S$ tends to Gaussian with mean 0 and variance $\sigma^2_X$ in the