Homework Set 5  
EECS 501  
Fri., Oct. 18, 2002

(in class, or by 3:30 PM in box in Room 4230)

Read Chapter 2 Sections 2.2, 2.3, and pp. 299, 323, 324, 330.

Topics: moments, joint moments, uncorrelated r.v's, independent r.v.'s, Markov and Chebychev inequalities, weak law of large numbers, convergence of random variables, entropy.

1. For each of the joint pmf's shown in the three tables below, find $\Pr(X \geq Y)$ and determine if $X$ and $Y$ are uncorrelated and/or independent.

<table>
<thead>
<tr>
<th>Y</th>
<th>X</th>
<th>Y</th>
<th>X</th>
<th>Y</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>1/6</td>
<td>0</td>
<td>1/6</td>
<td>-1</td>
<td>1/9</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1/3</td>
<td>0</td>
<td>0</td>
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<tr>
<td>1</td>
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<td>0</td>
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<td>1</td>
<td>1/9</td>
</tr>
</tbody>
</table>

   (a)   (b)   (c)

2. Problem 35, p. 79, Gubner
3. Problem 42, p. 81, Gubner
4. Problem 45, p. 81, Gubner
5. Problem 46, p. 81, Gubner

6. Let $X$ and $Y$ be uncorrelated random variables. Does $\text{var}(aX-bY) = \text{var}(aX)-\text{var}(aY)$? Prove or give a counterexample.

7. Let $X_1, X_2, ...$ be uncorrelated random variables, each with mean $m$ and variances $\sigma^2$ (both finite). Prove the following version of the weak law of large numbers.

   $\Pr\left( \left| \frac{1}{n} \sum_{i=1}^{n} X_i - m \right| \geq \frac{1}{n^{1/3}} \right) \leq \frac{\sigma^2}{n^{1/3}}$

8. (a) Define what it means for random variables $X_n$ to converge in probability to random variable $Y$.
   (b) Define what it means for random variables $X_n$ to converge in mean square to random variable $Y$.
   (c) Show that if $X_n$ converges in mean square to $Y$, then $X_n$ converges in probability to $Y$. Assume $\mathbb{E}[Y^2] < \infty$ and assume there is a constant $0 < M < \infty$ such that $\mathbb{E}[(X_n)^2] < M$ for all $n$. Hint: Use Chebychev's inequality.

9. Suppose $X$ and $Y$ are discrete random variables for which there are functions $f(x)$ and $g(x)$ such that $p_{XY}(x,y) = f(x) g(y)$ for all $x,y$.
   (a) Show that $X$ and $Y$ are independent random variables.
   (b) How are $f(x)$ and $g(y)$ related to the pmfs $p_X(x)$ and $p_Y(y)$?