Read: Section 6.3


1. Let $X_n$ be an IID random process with mean zero and variance $\sigma_X^2$. Consider the random process defined by $Y_n = X_n + X_{n-1}$, which is a stationary random process. Is $Y_n$ "mean ergodic"? "Mean ergodicity" is a weak kind of ergodicity, meaning that

$$\lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} Y_i = EX$$

You may use the fact that $X_n$ is ergodic in the full sense.

2. A wide-sense stationary random process $\{X(t)\}$ has power spectral density

$$S_X(f) = \frac{6f^2}{1+f^4}$$

Find the average power of the random process.

3. A zero-mean, white, Gaussian random process $X_t$ is input to a linear time-invariant system with impulse response

$$h(t) = \begin{cases} 
3 \ e^{-2t}, & t \geq 0 \\
0, & t < 0 
\end{cases}$$

Let $Y_t$ denote the output.

(a) Find the power spectral density of $Y_t$.
(b) Find the autocorrelation function of $Y_t$ and the variance of $Y_t$.
(c) Is $Y_t$ an independent increment process? Justify your answer.
(d) Is $Y_t$ a Markov process? Justify your answer.
(e) Find $\Pr(X_7 < 6)$.
(f) Compute the linear MMSE estimate of $X_7$ given $X_5=6$.
(g) Find $E(X_7-X_5)^2$