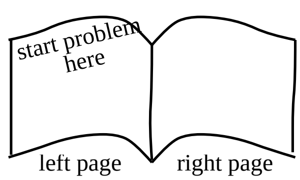
EECS 501 **Midterm 1** **Oct. 14, 1999**

Requirements

- 2 hours
- Closed book. You may bring one side of a page of notes.
- Calculators are permitted.
- Write your answers in a blue book or on separate sheets of paper. Do not write them on the exam questions and do not turn in the exam questions.
- You may use without rederivation the results derived in class, the book, the homework, handouts, or recitation.
- Write and sign the honor code pledge on your exam. ("I have neither given nor received aid on this exam, nor have I concealed any honor code violations.")
- Estimated points for each part of each problem are marked in square brackets. While grading, I sometimes find that changes are needed.

Testmanship:

- Problem 1 contains only short answer questions; no explanations or justifications are required.
- The other problems are more conventional. Although they don't require you to show your work or justify your approach, if you give a partially correct answer, you can get partial credit provided you show your work and I can see that you have a correct or partially correct approach. So it is recommended that you show your work and justify your approach.
- Don't try to cram too much on a page. Give yourself room to work.
- If you use a blue book, start each problem on a new "left" page. This gives you room to work, to make changes to previous problems, and to see (and check) as much of your solution at one time as possible.



- If you don't use a blue book, start each problem on a new page and write on only one side of a page. This gives you room to make changes to previous problems and to see (and check) as much of your solution at one time as possible. (You can't see both sides of a page at once.)

1. Short answer, definitional type questions. No explanations or elaborations are required.

[7] (a) **State** the "axioms of probability".

[6] (b) Let $\mathcal{B}$ be a collection of subsets of the sample space Ω. **Give** a brief precise definition of the "σ-algebra generated by $\mathcal{B}$".

[6] (c) Let $\Omega = (-\infty,\infty)$ be a sample space. **Give** a brief precise definition of the "Borel σ-algebra for Ω".

[6] (d) Let $A_1,\ldots,A_n$ be sets from the event space of some particular probability model. **What** does it mean for them to be "independent"? That is, give a brief precise definition.

[5] (e) Let X be a random variable. **Define** what it means for X to be "continuous". That is, give a brief precise definition. (You do not have to define "random variable".)

2.

[6] (a) Let $\Omega = \{1,2,3, ...\}$ and let $\mathcal{E}$ be the collection of all infinite subsets of Ω. **Is** $\mathcal{E}$ a σ-algebra?

[9] (b) Let $\Omega = (-\infty,\infty)$ be a sample space. **Which** of the following subsets of Ω are in the Borel σ-algebra for Ω?

$A = \{1, 4, 16, 32, \ \ldots \}$

$B = \{1, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ \ldots \}$

$J = [0,1] - \bigcup_{i=1}^{\infty} I_i$

where the I_i's are the intervals described in the definition of the Cantor distribution function. That is,

$I_1 = (\frac{1}{3},\frac{2}{3})$, $I_2 = (\frac{1}{9},\frac{2}{9})$, $I_3 = (\frac{7}{9},\frac{8}{9})$, $I_4 = (\frac{1}{27},\frac{2}{27})$, $I_5 = (\frac{7}{27},\frac{8}{27})$,

$I_6 = (\frac{18}{27},\frac{19}{27})$, $I_7 = (\frac{25}{27},\frac{26}{27})$, ...

3. The possible outcomes of a random experiment are a, 2, 3, and c . We are given that the probability of the set $A = \{a, 2\}$ is .4 and the probability of the set $B = \{a, 2, c\}$ is .6.

[8] (a) **Find** the σ-algebra generated by $\{A,B\}$.

[6] (b) **Find** the probability of those sets listed below that are in the σ-algebra generated by $\{A,B\}$.

$\{a, 3, c\}$, $\{3, a, 2\}$

[6] (c) **Find** the conditional probability of A given B.

4. It is known that the Russian space station Mir will eventually crash to Earth.

[10] (a) **Find** a probability model for the time of day at which it crashes. Assume that time is real number between 0 and 24. (Your model should not be unreasonable.)

[5] (b) **Find** two nonempty sets in your event space that are independent.

5. At random you choose one of the 70 slot machines in your favorite room of the downtown Detroit casino. Though you cannot tell from their appearance, 20 of the machines were made by company A and 50 by company B. Each machine has 5 independently spinning wheels, each of which eventually stops showing either a W or L. You win if exactly 3 of the 5 show W, otherwise you lose. The machines made by company A are "fair" in the sense that each wheel stops with a W showing with probablity 1/2. On the other hand, those made by company B are biased in such a way that the probability of a wheel stopping with W showing is 1/4.

[20] Given that you win the first time you play your chosen machine, **what is** the probability that it was made by company A?

It is sufficient to give a precise (and legible) expression for the answer; i.e you don't have to reduce it to a number, though doing so might enable you to check if it is a reasonable answer. Most solutions to this problem will involve making some intermediate deductions and finding some intermediate expressions. To increase your chance for partial credit in case of error, I recommend that you state them clearly.

6. A random variable X has probability distribution function $F_X(x)$, where

$$F_X(x) = \begin{cases} 0\ , & -1<x<0 \\ \frac{1}{10}e^{x}\ , & 0<x<1 \\ 1-\frac{1}{10}e^{-x}\ , & 1<x<\infty \end{cases}$$

[8] (a) **Find** $F_X(1)$ and $F_X(-2)$.

[6] (b) **What** type of random variable is X?

[6] (c) **Find** the probability that X is greater than 0 or less than 1.

7. Continuous random variables X and Y are independent. Each has probability density function equalling e^{-x} for $x\geq 0$ and 0 for $x< 0$. **Find** the probability that "X is between 1 and 2, and Y is between -1 and 3".

[10]

[130 points total]