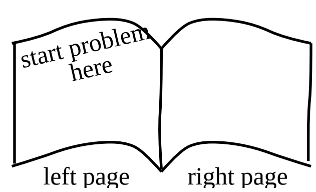
EECS 501 **Midterm 2** **Nov. 18, 1999**

Requirements

- 2 hours
- Closed book. You may bring two sides of a page of notes.
- Calculators are permitted.
- Write your answers in a blue book or on separate sheets of paper, not on the exam questions.
- Do not turn in the exam questions.
- If you use separate sheets of paper, add a cover sheet with your name on it.
- You may use without rederivation the results derived in class, the book, the homework, handouts, or recitation.
- Write and sign the honor code pledge on the cover of your exam. ("I have neither given nor received aid on this exam, nor have I concealed any honor code violations.")
- Estimated points for each part of each problem are marked in square brackets. While grading, I sometimes find that changes are needed.

Testmanship:

- Problem 1 contains only short answer questions; no explanations or justifications are required.
- The other problems are more conventional. Although they don't require you to show your work or justify your approach, if you give a partially correct answer, you can get partial credit provided you show your work and I can see that you have a correct or partially correct approach. So it is recommended that you show your work and justify your approach.
- Don't try to cram too much on a page. Give yourself room to work.
- If you use a blue book, start each problem on a new "left" page. This gives you room to work, to make changes to previous problems, and to see (and check) as much of your solution at one time as possible.



- If you don't use a blue book, start each problem on a new page and write on only one side of a page. This gives you room to make changes to previous problems and to see (and check) as much of your solution at one time as possible. (You can't see both sides of a page at once.)

1. Short answer questions. No explanations or elaborations are required.

[6] (a) **Give** an example of a pair of random variables (X,Y) such that each is continuous, but the pair does not have a joint probability density function.

[5] (b) **State** the condition that must be met in order that a pair of random variables (X,Y) has a joint probability density function.

[6] (c) **State** the defining formula for $\Pr(X\in A|Y=y)$ in the case that Y is a continuous random variable.

[5] (d) **State** the fundamental theorem of expectation for continuous random variables.

[7] (e) Random variables (X,Y) have joint probability density function $f_{XY}(x,y)$. **Find** a formula for $\Pr(X\in A|Y\in B)$ in terms of $f_{XY}(x,y)$, where A and B are Borel sets and $\Pr(Y\in B)>0$.

[5] (f) **State** a definition of the independence of random variables X and Y. Your definition should apply to all kinds of random variables.

[6] (g) If X is a discrete random variable, **state** the condition(s) that guarantees that E[X] exists.

2. Random variables X and Y have joint probability density function

$$f_{XY}(x,y) = 4\,xy, \quad 0 \le x \le 1, \quad 0 \le y \le 1,$$

[10] **Find** the expected value of X given Y = .9.

3. At the Probability Tutoring Center, it is found that customers who need a lot of help tend to arrive early. (The customers are smart!) Specifically, the arrival time T of the first customer is exponentially distributed with mean equal to 1 unit of time, and the amount of time H that a customer spends at the Center receiving help is exponentially distributed with mean equal to 1/T units of time.

[20] If the first customer receives help for exactly H = 2 units of time, **what** is the probability that he/she arrived before time 1?

You can receive full credit just by giving an integral expression for this probability, provided it contains only <u>one</u> integral and that the integrand and integral limits are simplified as much as possible.

Most solutions to this problem will involve making some intermediate deductions and finding some intermediate expressions. To increase your chance for partial credit in case of error, I recommend that you state them clearly.

4. A certain digital communication system sends an "a" with probability 1/4 and a "b" with probability 3/4. At the receiving end a random variable R is observed. When an "a" is sent, R is Gaussian distributed with mean 1 and variance 2; when a "b" is sent, R is again Gaussian distributed, but with mean 0 and variance 1.

[15] **Find** the variance of R.

5. Suppose X is uniformly distributed on the interval [0,4] and Y = g(X), where

$$g(x) = \begin{cases} 2x\ , & 0 \le x < 1/2 \\ 1\ , & 1/2 \le x < 1 \\ 4-2x\ , & 1 \le x \le 2 \\ 0\ , & \text{elsewhere} \end{cases}$$

[15] **Find** the probability distribution of Y. (You can describe the probability distribution in any way you choose.)

6. Let X and Y be independent random variables with X uniformly distributed between 0 and 1, and Y Gaussian distributed with mean 2 and variance 3.

Let U = 2X+3 and V = 4Y + 2.

[15] **Find** Pr(U≥4,V≤10).

[115 points total]