EECS 501 **Final Exam** **Dec. 16, 1999**

Requirements

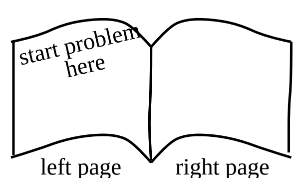
- 2 hours
- Closed book. You may bring three sides of a page of notes.
- Calculators are permitted.
- Write your answers in a blue book or on separate sheets of paper, not on the exam questions.
- If you use separate sheets of paper, add a cover sheet with your name on it.
- Do not turn in the exam questions.
- You may use without rederivation the results derived in class, the book, the homework, handouts, or recitation.
- Write and sign the honor code pledge on the cover of your exam. ("I have neither given nor received aid on this exam, nor have I concealed any honor code violations.")
- Estimated points for each part of each problem are marked in square brackets. While grading, I sometimes find that changes are needed.
- The error function table from Stark and Woods is attached to this exam.

Testmanship:

- Problem 1 contains only short answer questions; no explanations or justifications are required.
- The other problems are more conventional.

  Some require you to **justify** your answer. It should be as brief and precise as possible.

  Others don't require you to show your work or justify your approach. However, if you give a partially correct answer, you can get partial credit provided you show your work and/or justify your approch so that I can see that you have a correct or partially correct approach. Therefore, it is recommended that you show your work and justify your approach.
- Don't try to cram too much on a page. Give yourself room to work.
- If you use a blue book, start each problem on a new "left" page. This gives you room to work, to make changes to previous problems, and to see (and check) as much of your solution at one time as possible.



- If you don't use a blue book, start each problem on a new page and write on only one side of a page. This gives you room to make changes to previous problems and to see (and check) as much of your solution at one time as possible. (You can't see both sides of a page at once.)

1. Short answer questions. No explanations or elaborations are required.

[5] (a) Let $\Omega = [2,3]$ be a sample space. **Give** a brief precise definition of the "Borel σ-algebra for Ω".

[7] (b) In 25 words or less, **what** is the meaning of the "probability distribution" of a random variable X.

[5] (c) Let X be a random variable. **Define** what it means for X to be "discrete". That is, give a brief precise definition. (You do not have to define "random variable".)

[7] (d) **State** precisely the Weak Law of Large Numbers.

[6] (e) **Define** the (strict sense) "stationarity" property for a continuous-time random process $\{X(t): -\infty<t<\infty\}$.

2. I spray my rosebush with pesticide on 100 successive days. Each time I spray, the amount of pesticide that reaches the bush is uniformly distributed between 0 and 3, independent of other sprayings. If at the end of the 100 days, the total amount of pesticide received by the bush is more than 165, the pesticide will kill the bush. If the total pesticide is less than 135, the bugs will kill the bush. ATTACH ERROR FUNCTION TABLES

[15] **Find**, approximately, the probability that the rosebush lives. *(from 501, F91, Ex. 2, Pr. 5)*

3. Let $X = Y + Z$ where Y and Z are IID random variables and Y has probability density *(from 501, F91, Ex. 2, Pr. 6*

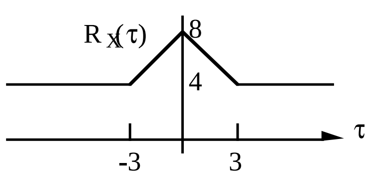
$$f_Y(y) = \begin{cases} e^{-y}, & y \ge 0 \\ 0, & \text{else} \end{cases}$$

[10] (a) **Find** the conditional density of X given Y=y. **Simplify** as much as possible.

[15] (b) **Find** the minimum mean squared error estimator for Y based on X. **Simplify** as much as possible.

(If you had difficulty with Part (a) and believe that you need to use the answer to (a) to solve (b), then you can get substantial partial credit for (b) by giving an expression for the desired estimator in terms of the conditional density for X given Y and whatever else is needed. Simplify as much as possible.)

4. A continuous-time wide-sense stationary random process $\{X(t)\}$ has mean function $\mu_X(t) = 2$ and autocorrelation function $R_X(\tau)$ shown below:



[8] (a) **Find** two times t_1 and t_2 such that $X(t_1)$ and $X(t_2)$ are uncorrelated. **Justify** your answer by demonstrating the necessary uncorrelation.

[7] (b) **Is** the random process mean ergodic? **Justify** your answer.

5. Suppose that with probability one the sample functions x(t) from a stationary, ergodic, continuous-time Gaussian random process {X(t)} satisfy

$$\frac{1}{T}\int_0^T x(t)x(t+3)\,dt \to 9,\quad \frac{1}{T}\int_0^T x(t)\,dt \to 3,\quad \text{and}\quad \frac{1}{T}\int_0^T x^2(t)\,dt \to 10,\ \text{ all as } T \to \infty.$$

[8] (a) **Are** X(7) and X(4) independent random variables? **Justify** your answer.

[11] (b) **Find** Pr(0<X(7)-X(4)<2) **attach error function tables**

[6] (c) Suppose we had not been told explicity that the random process is ergodic. **Could** we conclude from the other "givens" that the process is mean ergodic? **Could** we conclude that it is ergodic?

6. For a certain simulation project we desire to create a stationary, continuous-time Gaussian random process {X(t)} with power spectral density

$$S_X(\omega) = \begin{cases} 8\pi e^{-5|\omega|}, & |\omega|<1 \\ 0, & \text{else} \end{cases}$$

Suppose we have available a device that emits a wide-sense stationary, continuous-time Gaussian random process {Y(t)} with power spectral density

$$S_Y(\omega) = \begin{cases} 3e^{-|\omega|}, & |\omega|<2 \\ 0, & \text{else} \end{cases} .$$

[15] **Describe** how one can create the desired random process from the given one, with no sources of randomness other than {Y(t)}. Make sure your description is complete. But you need not describe its implementation.

7. Let $(\Omega_U, \mathcal{E}_U, P_U, U)$ be an underlying probability space, where $\Omega_U = [0,2\pi]$, $\mathcal{E}_U$ = Borel σ-algebra on Ω_U, and P_U is the probability measure defined by $P_U((a,b)) = (b-a)/2\pi$. Let {X(n): n=1,2,...} be the discrete-time random process defined by the functions

$$X(n,u) = \sin nu, \quad n = 1,2,\ldots, \quad u \in \Omega_U$$

[5] (a) **Find** the mean function.

[10] (b) **Find** the autocorrelation function.

[5] (c) **Is** the random process wide-sense stationary? **Justify** your answer.

[145 Total Points]