1. Consider a pair of random variables $X, Y$ modelled as

$$\Omega_u = [0,1], \quad \mathcal{E}_u = \text{Borel } \sigma\text{-alg}, \quad \text{and } P_u(F) = \text{Borel measure}$$

$$X(\omega) = \omega^2, \quad Y(\omega) = \begin{cases} 1, & 0 \leq x \leq 0.5 \\ 0, & 0.5 < x \leq 1 \end{cases}$$

(a) Find $\Pr(X \geq 0.5)$.

(b) Find $\Pr(Y > 0.2)$

(c) Find $\Pr(X < 0.3, Y < 0.3)$

(d) Are $X$ and $Y$ independent? Prove or disprove.

2. Consider the underlying probability model $$(\Omega_u, \mathcal{E}_u, P_u, U)$$ with

$$\Omega_u = [0,1], \quad \mathcal{E}_u = \sigma\{ (0.1,0.3), [0.3,0.7), [0.7,1]) \} \quad \text{and} \quad P_u(F) = \text{length of } F \text{ for } F \in \mathcal{E}_u.$$

Which of the following functions are measurable with respect to $\mathcal{E}_u$.

(a) $X(\omega) = \omega + 7s$

(b) $X(\omega) = \begin{cases} 2, & 0 \leq \omega \leq 0.5 \\ 3, & 0.5 < \omega \leq 1 \end{cases}$

(c) $X(\omega) = \begin{cases} 1, & 0 \leq \omega < 0.7 \\ 3, & 0.7 \leq \omega \leq 1 \end{cases}$

(d) $X(\omega) = \begin{cases} 4, & 0 \leq \omega < 0.3 \\ \pi, & 0.3 \leq \omega \leq 0.7 \\ -1, & 0.7 < \omega \leq 1 \end{cases}$

3. Consider the random experiment described in Prob. 44 and Fig. 1.11 on page 55 of Gubner. Let $T$ be a random variable representing the input to the channel, and let $R$ be a random variable representing the output of the random variable.

(a) Are the random variables discrete, continuous or mixed?

(b) Find a direct model of the pair of random variables $T$ and $R$.

(c) Find the probability mass functions of $T$ and $R$.

(d) Find the joint probability mass function for this pair of random variables. (Make a table giving the values of the pmf.)

(e) For what values of $\epsilon$ and $\delta$ are $T$ and $R$ independent?

(f) Find a functional model for this pair of random variables. (The random variables are to be modelled as functions defined on a common underlying probability model.) Be sure to verify that your model induces the joint probability mass function. Also, make sure that measurability is satisfied.)