1. Prob. 28, p. 114, Gubner.


4. Let $X_1, X_2, \ldots$ be uncorrelated random variables, each with mean $m$ and variances $\sigma^2$ (both finite).

Prove the following version of the weak law of large numbers.

$$\Pr\left( \left| \frac{1}{n} \sum_{i=1}^{n} X_i - m \right| \geq \frac{1}{n^{1/3}} \right) \leq \frac{\sigma^2}{n^{1/3}}$$

5. (a) Define what it means for random variables $X_n$ to converge in probability to random variable $Y$.

(b) Define what it means for random variables $X_n$ to converge in mean square to random variable $Y$.

(c) Show that if $X_n$ converges in mean square to $Y$, then $X_n$ converges in probability to $Y$. Assume $E[Y^2] < \infty$ and assume there is a constant $0 < M < \infty$ such that $E[(X_n)^2] < M$ for all $n$. Hint: Use Chebychev's inequality.


7. Prob. 65, p. 119, Gubner.

8. Prob. 68, p. 120, Gubner. Simplify the answer until you can recognize the answer as well known kind of pmf.

9. Prob. 70, p. 120, Gubner.

10. Prob. 77, p. 120, Gubner. (Hint: Use conditional expectation)

11. (a) Show that if $E[Y|X=x] = E[Y]$ all $x$, then $X$ and $Y$ are uncorrelated. (Hint: Substitution and total expectation laws.)

(b) Give a counterexample, to show that the converse is not true.

12. Suppose that the number of accidents that occur in a week at an industrial plant is a random variable $A$; suppose that the number of workers injured in the $ith$ accident is a random variable $N_i$ with mean $m$ (same for all $i$); and assume that the number of workers injured in an accident is independent of the number that occur in the week.

Show that the expected number of injured workers in one week is $E[A] m$. (Hint: First find a formula for the number of injured workers in one week. Then use the substitution and total expectation laws.)