1. By direct integration, find the Fourier transform of the signal

\[ x(t) = \begin{cases} 
\cos 2\pi f_o t, & -\frac{T}{2} \leq t \leq \frac{T}{2}, \\
0, & \text{else,}
\end{cases} \]

where \( f_o > 0 \).

Simplify as much as possible.

2. A wide-sense stationary random process \( X_t \) has power spectral density

\[ S_{mX}(f) = \frac{6f^2}{1+f^4} \]

Find the average power of the random process.

3. A zero-mean, white, Gaussian random process \( X_t \) with power spectral density \( S_X(f) = 4, \) for all \( f, \) is input to a linear time-invariant system with impulse response

\[ h(t) = \begin{cases} 
3e^{-2t}, & t \geq 0, \\
0, & t < 0
\end{cases} \]

Let \( Y_t \) denote the output.

(a) Find the power spectral density of \( Y_t. \)

(b) Find the autocorrelation function of \( Y_t \) and the variance of \( Y_t. \)

(c) Find \( \Pr(Y_T < 6). \)

(d) Compute the linear MMSE estimate of \( Y_T \) given \( Y_5 = 6. \)

(e) Find \( E(Y_T - Y_5)^2. \)

4. Let \( X_t \) and \( Y_t \) be independent wide-sense stationary random processes. ("Independent" means any finite collection of the \( X_t \)'s are independent of any finite collection of the \( Y(t)'s). \) Let

\[ Z(t) = X(t)Y(t) \]

(a) Show that \( \{Z(t)\} \) is WSS.

(b) Find the power spectral density of \( Z_t \) in terms of the power spectral densities of \( X_t \) and \( Y_t. \)