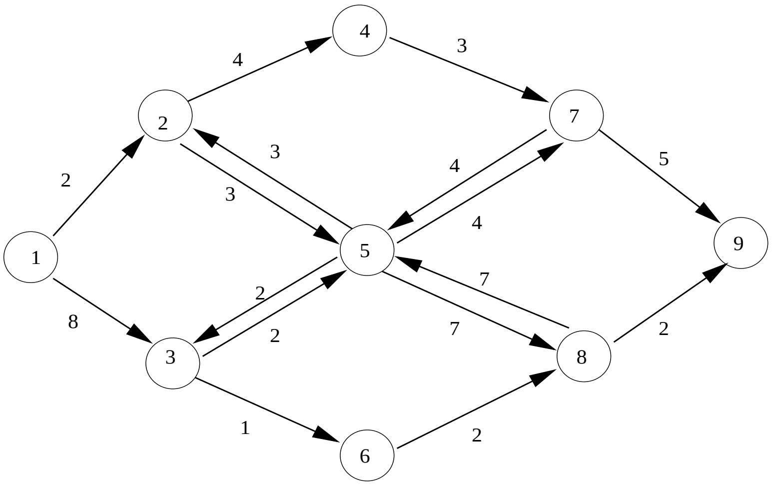
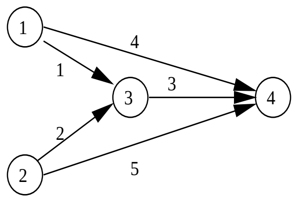
**EECS 509 – WINTER 1997**

**Problem Set #10 – Not to be turned in – Will be discussed in last class**

1. Find a shortest path from node 1 to node 9 in the network shown below (considered in Problem Set 9) assuming that there is a penalty of 5 units of cost for *turning right* at nodes 2, 5, and 7.




2. Consider the network below. There are two O-D pairs, (1, 4) and (2, 4). The five links are numbered as indicated in the figure.




The link costs (per vehicle), $c_i(f_i)$, depend on flows as follows:

$$\begin{aligned} c_1(f_1) &= 2 + f_1^2 \\ c_2(f_2) &= 3 + f_2 \\ c_3(f_3) &= 1 + 2f_3^2 \\ c_4(f_4) &= 2 + 4f_4 \\ c_5(f_5) &= 3 + 4f_5 \ . \end{aligned}$$

The flow demands for the O-D pairs are 5 for (1, 4) and 6 for (2, 4).

(a) Write the flow conservation equations, $E\underline{f}^\alpha = \underline{g}^\alpha$, for each origin node $\alpha$.

(b) Write the path-flow to link-flow equations, $A^{(k)}\underline{h}^{(k)} = \underline{f}^{(k)}$, for each O-D pair $k$.

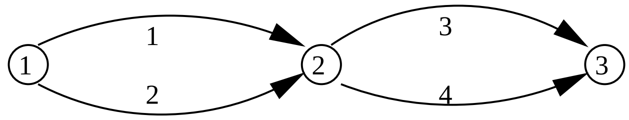
(c) Pick an admissible path flow assignment $\underline{h}$.

i. Verify that the flow conservation equations are satisfied for this $\underline{h}$.

ii. Calculate the total network cost $C$.

iii. Is your assignment system optimal? Does it lead to user equilibrium?

3. Consider the network below. There is one O-D pair, (1, 3). The four links are numbered as indicated in the figure.




The link costs (per vehicle), $c_i(f_i)$, depend on flows as follows:

$$\begin{aligned} c_1(f_1) &= 2 + f_1^2 \\ c_2(f_2) &= 3 + f_2 \\ c_3(f_3) &= 1 + 2f_3^2 \\ c_4(f_4) &= 2 + 4f_4 \,. \end{aligned}$$

The flow demand for the O-D pair is 4.

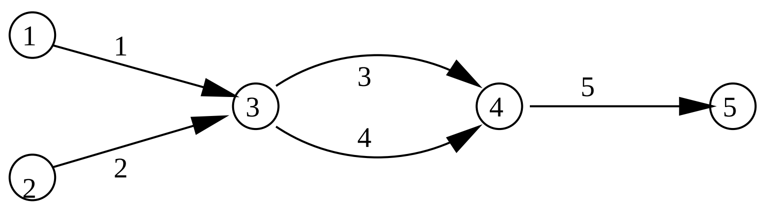
(a) Find the system optimal link flows and the corresponding total cost $C$.

(b) Find the user equilibrium link flows and their associated travel times.

(c) Compare the SO and UE solutions.

*Hint: Decompose the problem into two sub-problems...*

4. Consider the network below. There are two O-D pairs, (1, 5) and (2, 5). The five links are numbered as indicated in the figure.




The link costs (per vehicle), $c_i(f_i)$, depend on flows as follows:

$$\begin{aligned} c_1(f_1) &= 1 \\ c_2(f_2) &= 2 \\ c_3(f_3) &= 2 + f_3 \\ c_4(f_4) &= 1 + 2f_4 \\ c_5(f_5) &= 1 \,. \end{aligned}$$

The flow demands for the O-D pairs are 2 for (1, 5) and 3 for (2, 5).

(a) Find the user equilibrium link flows and their associated travel times.

(b) Are the user equilibrium path flows unique?

(c) Do the user equilibrium link flows achieve system optimality?

*Hint: Determine $f_1$, $f_2$, and $f_5$ by inspection.*