EECS 509 – WINTER 1997

Problem Set #1 – Assigned Jan. 16 – Due in class Jan. 23

1. Problem 1.2 in Sheffi. (See handout.)

   As this material was not covered in class, read section 1.3 in Sheffi.

2. Time mean speed vs. space mean speed.

   (a) Build an example using a single (non-looping) link where the space mean speed is strictly less than the time mean speed. Use deterministic flows. (Recall the discussion in class.)

   (b) Wardrop showed in 1952 that

   $$\overline{u_t} = \overline{u_s} + \frac{\sigma_s^2}{\overline{u_s}}$$

   where the "variance" σ_s^2 is defined as

   $$\sigma_s^2 = \frac{1}{k}\sum_i k_i(u_i - \overline{u_s})^2$$

   where i ranges over all the traffic streams on the link, k_i is the density of stream i, and u_i the speed of stream i.

   i. Verify that Wardrop's relationship holds in your example above.

   ii. (OPTIONAL) Demonstrate Wardrop's relationship. Note that the fundamental equation holds for each individual traffic stream as well, i.e., $q_i = k_i u_i$, where q_i is the flow of stream i.

3. A section of highway has a free flow speed of 55 mph and a jam density of 367 vpm. Assume a linear speed-density relationship (e.g., Greenshield's model). Calculate the density at maximum flow, the maximum flow rate, and the speed at maximum flow.

4. The speed-density relationship for a certain road segment is:

   $$\overline{u_s} = \begin{cases} u_f & \text{if } k \leq k_0 \\ u_f\frac{k_j - k}{k_j - k_0} & \text{if } k_0 < k < k_j \\ 0 & \text{if } k \geq k_j \end{cases}$$

   where the notation is as in class; k_0 corresponds to the maximum uncongested density.

   (a) Plot the "fundamental diagram" flow vs. density for this road.

   (b) Find the road's capacity and the speed at capacity flow.

   Hint: Consider the two cases $k_0 \leq \frac{k_j}{2}$ and $k_0 > \frac{k_j}{2}$ separately.