EECS 509 – WINTER 1997

Problem Set #3 – Assigned Jan. 30 – Due in class Feb. 6

1. Derive the expression of the density wave velocity, c, in the case of Greenberg's model. What is the density wave speed relative to the vehicle speed, u? Can you give a physical interpretation of your answer?

2. Consider the situation of an initial density $k(x, 0)$ equal to k_j for $x < 0$ and to 0 for $x > 0$. This can be interpreted as a long queue of vehicles waiting at a red light, with the light turning green at $t = 0$. We are interested in how the density will distribute as t increases. (We pretend that k is continuous around $x = 0$ at $t = 0$.)

   Assuming Greenshield's model, it can be shown that the solution of the continuity equation (i.e., the law of conservation of vehicles) for this initial condition $k(x, 0)$ is

   $$k(x,t) = \frac{k_j}{2}(1 - \frac{x}{u_f t}) \text{ for } t > 0.$$

   (a) Verify that this expression does indeed satisfy the continuity equation.

   (b) Plot $k(x, t)$ as a function of x for a fixed $t > 0$.

   (c) Verify that as $t \to \infty$, the density $k(x, t)$ tends to the density k_m (defined to be the density at capacity) for all x.

   (d) How long does it take for a vehicle at position $x = -x_0$ in the queue to start moving after the light turns green?

3. Read Chapters 1 and 2 of the INTEGRATION 2.0 User's Guide and then run Demonstrations A and B to familiarize yourself with this simulator.

   Also, read Chapters 3 and 4 of the User's Guide to familiarize yourself with the input and output files.