EECS 509 – WINTER 1997

Problem Set #7 – Assigned Mar. 11 – Due in class Mar. 20

1. Consider the algorithm of Brooks studied in class for maximizing the green band on an arterial. Once it has been determined if a left or right interference should be chosen for a given intersection, the actual offset must be determined. Using the same figures that we used to establish the expressions of l_i and $\tilde{r}_i$, *explain* the following "rules":

   - If m_i is *even* and you want *right interference*, then align the green phase of intersection i over the green phase of intersection 0.
   - If m_i is *even* and you want *left interference*, then align the red phase of intersection i over the green phase of intersection 0.
   - If m_i is *odd* and you want *right interference*, then align the red phase of intersection i over the green phase of intersection 0.
   - If m_i is *odd* and you want *left interference*, then align the green phase of intersection i over the green phase of intersection 0.

2. Consider the example studied in class of an arterial with five signalized intersections with cross streets. Let the (new) cycle length be $c = 86$ sec., which includes 6 sec. of lost time. Let all the other data, including the splits (in %), be the same as in the example done in class.

   Find b^* using the algorithm of Brooks and specify which intersections will have left interferences and which will have right interferences. Find the corresponding offsets for the four intersections with respect to intersection 0. Draw the picture of these initial offsets.

3. Consider a discrete-time model for dynamic control of signalization at a two-phase intersection of two one-way streets with constant demand. The discrete time index k corresponds to a time interval of some duration, say x seconds. (The value of x is not needed for what follows.) The green phase may be changed at the beginning of each such time interval. For simplicity, we neglect lost times due to switching. We have the following information:

   - On approach one, served by phase 1, 4 vehicles arrive during each time interval.
   - On approach two, served by phase 2, 5 vehicles arrive during each time interval.
   - When phase 1 is green, up to 6 vehicles may leave during one time interval.
   - When phase 2 is green, up to 15 vehicles may leave during one time interval.
   - Let $q_i(k)$ be the queue length on approach i at the beginning of the kth time interval. The initial conditions are $q_1(0) = q_2(0) = 0$.

- Let $Q_1(k) = q_1(k) + 4$ and $Q_2(k) = q_2(k) + 5$, i.e., $Q_i(k)$ is the sum of the queue length at approach i at the beginning of the kth time interval plus the number of arrivals on approach i during that time interval.

(a) Suppose that the control policy that is adopted is as follows: at time index k, serve (i.e., give the green to) the phase with the largest value of $Q_i(k)$. We call this policy "serve the longest queue."

Calculate the signal timing plan that results from this policy with the above data and verify that it converges to a periodic behavior.

In each "period", how many green intervals are given to phase 1 and how many are given to phase 2?

(b) Suppose now that the policy that is adopted is to serve at time index k the phase that would lead to the largest number of vehicle departures in the kth time interval. We call this policy the "maximum instantaneous throughput policy."

Verify that in this example, this policy is *unstable* (i.e., one of the queues will grow arbitrarily large).

(c) Explain why with the above data "serve the longest queue" is stable while "maximum instantaneous throughput" is not by computing the corresponding *degrees of saturation* for each approach in each case.

Note: In (a) and (b), ties are resolved by either choice.