EECS 509 – WINTER 1997

Problem Set #8 – Assigned Mar. 20 – Due in class Mar. 27

1. *How long should a traffic light remain yellow?*

   A driver who sees a light turning yellow has two choices: brake to a complete stop or pass through the intersection. Assume that the driver is travelling at the speed limit, denoted u_f. If she decides to stop, then she must have an adequate distance to do so; denote this minimum distance by D_b. If she decides to pass through, then she must have adequate time to pass completely through the intersection, including: (i) a time interval in which to decide (reaction time, denoted T), (ii) the time it takes to drive (at speed u_f) the minimum distance needed to stop, D_b, and (iii) the time it takes to drive through the intersection (still at speed u_f), equal to $(I + L)/u_f$, where I is the width of the intersection and L is the length of the vehicle (note that the rear end of the vehicle must clear the intersection). Thus the yellow phase should be long enough to allow for the second alternative, where the driver decides to pass through. The rationale for this is that a driver should never be in the dilemma of being too close to stop and yet too far to completely pass through before the light turns red.

   (a) Given the above, find an expression for the length of the yellow phase, denoted Y, in terms of the above parameters and in terms of g, the gravitational acceleration, and f, the coefficient of friction. Consider the following model for braking:

   $$\frac{W}{g}\frac{d^2x(t)}{dt^2} = -fW$$

   where W is the weight of the vehicle and x its position.

   (b) Plot Y vs. u_f using typical values for the necessary parameters (take $f = 0.2$).

2. Recall the example of real-time control of an intersection of two one-way streets solved in class with three arrivals from the west at times $t = 3$, 5, and 9 sec. and one arrival from the north at time $t = 6$ sec. Suppose that the vehicle arriving from the north is a bus. Since the bus carries more passengers, we decide to give more weight to delay of the bus in the evaluation of the arc costs in the calculation of the optimal solution.

   What minimum weight is needed to give "priority" to the bus, i.e., the bus gets the green first, not the vehicles arriving from the west. For your calculations, use the same data as used in class. Also, consider only integer weigths.

   Can you find a different initial state $x(0)$ that requires less weight but produces the same desired result?

(over)

3. Rolling-horizon implementations may not necessarily be "globally optimal", i.e., the "pasting" of optimal solutions to problems with limited horizons may not be optimal when solving the optimization problem for the complete horizon (assuming this is feasible). Build a *simple* example where this happens. If you wish, you can start from the example used in Problem 2 (here, all vehicles have equal weight with respect to delay) and add arrivals. You can use partial roll or full roll. You can limit your example to one roll if you wish. Show all your calculations.