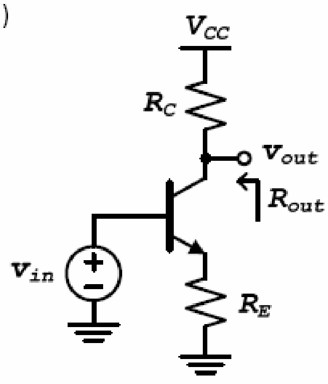
Problem 1.1: For each of the following circuits, find simplified expressions for gain ($A_v = v_{out} / v_{in}$) and R_{out}. Solving by inspection is preferred. It is ok to keep parallel combinations of resistors in the form $(R_1 || R_2)$. For simplicity, assume that g_m, r_o, and R_π are identical for all devices, ignore body effect, and ignore r_o when appropriate ($1/g_m$, R_C and $R_E << r_o$).

a)



$G_m \equiv$ effective g_m

$$G_m = \frac{\partial I_C}{\partial V_{in}} = \frac{\partial I_C}{\partial V_{BE}} \cdot \frac{\partial V_{BE}}{\partial V_{in}} = g_m \frac{\partial V_{BE}}{\partial V_{in}}, \quad V_{BE} = V_{IN} - \alpha I_C R_E$$

$$G_m = g_m \frac{\partial}{\partial V_{IN}}\left[V_{IN} - \alpha I_C R_E\right] = g_m\left[1 - \alpha R_E \frac{\partial I_C}{\partial V_{IN}}\right]$$

$$G_m = g_m\left[1 - \alpha R_E G_m\right] \Rightarrow G_m\left[1 + \alpha R_E g_m\right] = g_m$$

$$G_m = \frac{g_m}{1 + \alpha g_m R_E}, \quad \alpha \approx 1$$

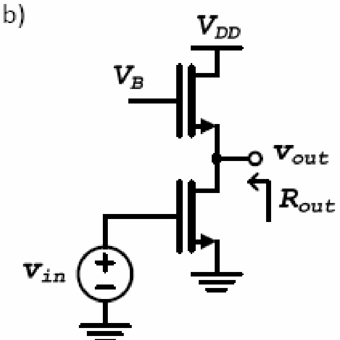
$$G_m \approx \frac{g_m}{1 + g_m R_E}$$

$\boxed{R_{OUT} \approx R_C}$ since $r_o >> R_C$

$$A_V = -G_m R_{OUT} = -\frac{g_m}{1 + g_m R_E} R_C$$

$$\boxed{A_V = -\frac{g_m R_C}{1 + g_m R_E}}$$

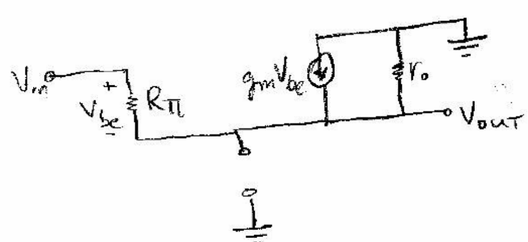
b)



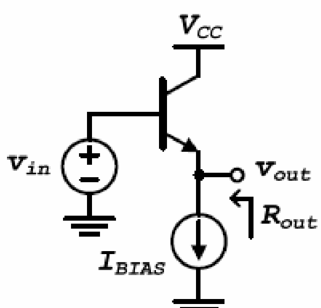
$$R_{OUT} = \frac{1}{g_m} || r_o \approx \boxed{\frac{1}{g_m}}$$

$$A_V = -g_m R_{OUT} \approx -g_m\left(\frac{1}{g_m}\right)$$

$$\boxed{A_V \approx -1}$$



c)



$$V_{be} = V_{in} - V_{OUT}$$

$$V_{OUT} = g_m V_{be} r_o = g_m r_o (V_{in} - V_{OUT})$$

$$V_{OUT}(1 + g_m r_o) = V_{in} g_m r_o$$

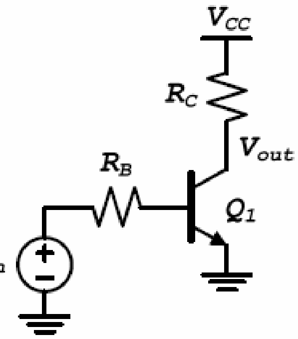
$$A_V = \frac{V_{OUT}}{V_{in}} = \frac{g_m r_o}{1 + g_m r_o}, \quad g_m r_o >> 1$$

$$\boxed{A_V \approx 1}$$

$$R_{OUT} = R_\pi || \frac{1}{g_m} || r_o, \quad r_o >> R_\pi >> \frac{1}{g_m}$$

$$\boxed{R_{OUT} \approx \frac{1}{g_m}}$$

Problem 1.2: Use the circuit on the right for this problem. Include base width modulation in your calculations. Assume $\beta_F = 120$, $V_A = 35V$, $V_{CE(sat)} = 0.3V$, and $V_{BE} = 0.7V$ when Q_1 is in the forward active region.



a) Find the DC values of I_B, I_C, and V_{out} at the quiescent point. Assume $V_{IN} = 2.7V$, $R_B = 100k\Omega$, $V_{CC} = 5V$, and $R_C = 1k\Omega$. What region is Q_1 operating in?

$$I_B = \frac{V_{IN} - V_{BE}}{R_B} \qquad I_C = \beta I_B\left(1 + \frac{V_{CE}}{V_A}\right) \qquad I_C = \frac{V_{CC} - V_{OUT}}{R_C}$$

$$I_B = \frac{2.7 - 0.7}{100K} = \boxed{20\mu A}$$

$$\beta I_B\left(1 + \frac{V_{OUT}}{V_A}\right) = \frac{V_{CC} - V_{OUT}}{R_C}, \quad V_{OUT} = V_{CE}$$

$V_A = 35V$
$V_{CC} = 5V$
$\beta = 120$
$R_C = 1K\Omega$
$R_B = 100k\Omega$

$$V_{OUT} = V_A\left[\frac{V_{CC} - \beta I_B R_C}{\beta I_B R_C + V_A}\right]$$

$$\boxed{V_{OUT} \approx 2.433\ V}$$

$$\boxed{I_C = \frac{V_{CC} - V_{OUT}}{R_C} = 2.567\ mA}$$

Since $V_{CE} = V_{OUT} \approx 2.433 > V_{CE(sat)}$ $\therefore$ foward active

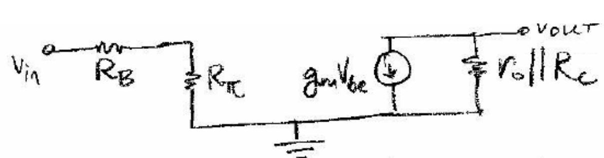
b) Find the values of g_m, R_π, and r_o at the quiescent point found in part a) and room temperature.

$$g_m = \frac{I_C}{V_T} \approx \frac{2.567m}{26m} \approx \boxed{98.7\ mS}$$

$$R_\pi = \frac{\beta}{g_m} = \frac{120}{98.7m} \approx \boxed{1.216\ k\Omega}$$

$$r_o = \frac{V_A}{I_C} = \frac{35}{2.567m} \approx \boxed{13.365\ k\Omega}$$

c) Derive a small signal expression for v_{out}/v_{in}, you do not need to solve for the numerical value.

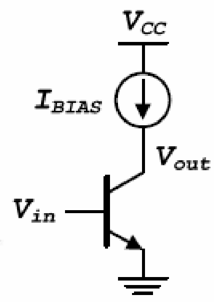


$$V_{OUT} = -g_m V_{be}\left(r_o \| R_C\right), \qquad V_{be} = V_{in}\left(\frac{R_\pi}{R_\pi + R_B}\right)$$

$$V_{OUT} = -g_m V_{in}\left(\frac{R_\pi}{R_\pi + R_B}\right)\left(r_o \| R_C\right)$$

$$\boxed{A_v = \frac{V_{OUT}}{V_{in}} = -g_m\left(r_o \| R_C\right)\left(\frac{R_\pi}{R_\pi + R_B}\right)}$$

Problem 1.3: Use the circuit on the right for this problem. Assuming the transistor is biased in the forward active region, calculate the numerical value for small signal gain given V_A = 35V. Assume room temperature. This is the intrinsic gain of the device.

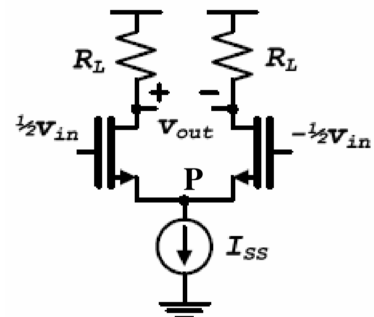


$I_{Bias} = I_C = \beta I_B = \beta \frac{V_{IN}}{R_\pi}$ $\quad I_c = g_m V_T \quad R_\pi = \frac{\beta}{g_m}$

$V_{OUT} = (I_c)(r_o) = \left(\beta \frac{V_{IN}}{R_\pi}\right)\left(\frac{V_A}{I_c}\right) = \left(\frac{\beta V_{in}}{\beta/g_m}\right)\left(\frac{V_A}{g_m V_T}\right) = V_{in}\left[\frac{V_A}{V_T}\right]$

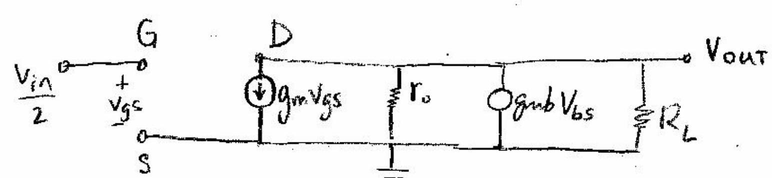
$$\boxed{A_v = \frac{V_{OUT}}{V_{in}} = \frac{V_A}{V_T} = \frac{35}{26m} \approx 1346.15 \ V/A}$$

Problem 1.4: Use the circuit on the right for this problem. Assume the circuit is symmetric.

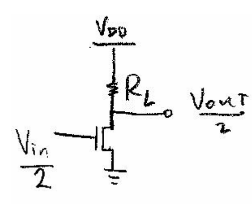


a) Draw the small signal half-circuit of the diff pair. Include channel length modulation and body effect in your model.

* Point P looks like AC gnd.



b) Find an expression for gain v_{out}/v_{in}.



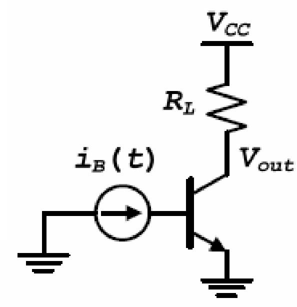
$A_v = -g_m (R_L || r_o)$

$$\boxed{A_v \approx -g_m R_L}$$

Problem 1.5: For this problem, use the parameter values given in the table below. Use the charge control model to solve the following parts.

Parameter	NPN	Units
β_F	100	A/A
β_R	10	A/A
τ_F	10	ps
τ_R	5	ns
$V_{CE(sat)}$	0	V

a) For the circuit on the right, assume i_B is initially 0, and steps from 0 to 10μA at time 0s and remains at 10μA. V_{CC} = 5V and R_L = 2kΩ. What region will the transistor be in at t = ∞? Calculate the final values of q_F (in Coulombs), i_C, and v_{OUT}.



$V_{CC} = 5V \quad R_L = 2k\Omega \quad I_B - 0 - 10\mu A$

$i_C = \beta_F I_B \left(1 - e^{-t/\tau_{BF}}\right)$

At $t = \infty \longrightarrow i_C = \beta_F I_B \left(1 - e^{-\infty}\right) = \beta_F I_B$

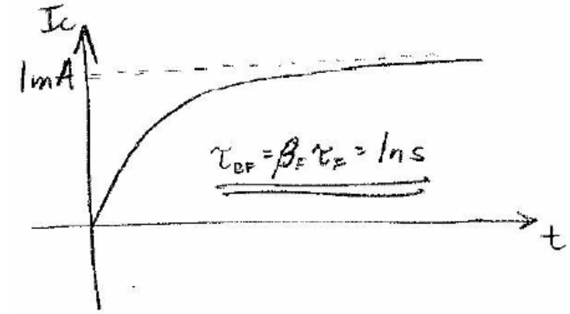
$i_C = 100\,(10\mu A) = \boxed{1mA}$

$q_F = i_C \tau_F = (1m)(10p) = \boxed{1 \times 10^{-14}\ C}$

$V_{OUT} = V_{CC} - I_C R_L = 5 - (1m)(2k) = \boxed{3V}$

$\therefore$ $\boxed{\text{foward active}}$

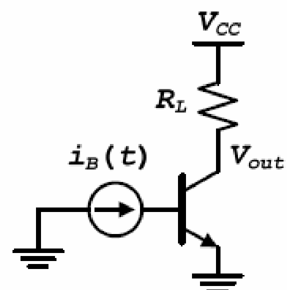
b) Sketch the transient current i_C from part a) as a function of time. What is the time constant of the response?



Problem 1.6: For this problem, use the parameter values given in the table below. Use the charge control model to solve the following parts.

Parameter	NPN	Units
β_F	100	A/A
β_R	10	A/A
τ_F	10	ps
τ_R	5	ns
$V_{CE(sat)}$	0	V

a) Now assume i_B is initially 0, and steps from 0 to 100μA at time 0s and remains at 100μA. V_{CC} = 5V and R_L = 2kΩ. The BJT will obviously be in saturation at t = ∞. Using the charge control models in the saturation region, calculate the final values of $q_{TOTAL} = q_F + q_R$, i_C, and v_{OUT}.
Hint: I_C is determined by the circuit, and at time t = ∞ (steady state), all *dq/dt* terms equal 0.



$$I_C = \frac{V_{CC} - V_{CE(sat)}}{R_L} = \frac{5}{2k} = 2.5\,mA$$

At $t = \infty$

$$I_C = \frac{q_F}{\tau_F} - q_R\left[\frac{1}{\beta_R \tau_R} + \frac{1}{\tau_R}\right] = \frac{q_F}{10p} - q_R\left[\frac{1}{50n} + \frac{1}{5n}\right]$$

$$I_C = (.1 \times 10^{12})q_F - (.22 \times 10^{9})q_R = 2.5 \times 10^{-3} \qquad (1)$$

$$I_B = \frac{q_F}{\beta_F \tau_F} + \frac{q_R}{\beta_R \tau_R} = \frac{q_F}{1n} + \frac{q_R}{50n}$$

$$I_B = (10^9)q_F + (.02 \times 10^9)q_R = 100 \times 10^{-6} \qquad (2)$$

Solving equations (1) and (2) simultanously gives

$q_R = 3.378\,pC$ and $q_F = 0.032\,pC$

Then $q_{total} = q_F + q_R = \boxed{3.41\,pC}$

$I_C = i_c(t=\infty) = \boxed{2.5\,mA}$

$V_{OUT} = V_{CE(sat)} = \boxed{0\,V}$

b) At time t = 0 the base charge is 0 and the device is in the forward active region. Let's assume for now that the device will remain in forward active and not saturate. Calculate the final value of i_C and V_{out} using the *forward active* charge control models (V_{out} will be negative). Sketch V_{out} as a function of time, and indicate what the time constant of the response is. Calculate the time at which V_{out} reaches the saturation voltage 0V.

→ At $t = \infty$

$$I_B = 100\mu = \frac{q_F}{\tau_F \beta_F} = \frac{q_F}{1n} \longrightarrow q_F = 0.1pC$$

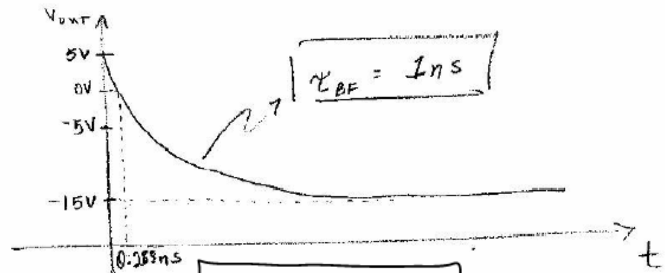
$$I_c = \frac{q_F}{\tau_F} = \frac{.1p}{10p} = 0.01 \longrightarrow \boxed{I_c = 10mA}$$

$$V_{OUT} = V_{cc} - I_c R_L = 5 - (10m)(2k) = \boxed{-15V}$$

→ To sketch V_{OUT}

$$V_{OUT} = V_{cc} - i_c R_L = V_{cc} - R_L \beta_F I_B \left(1 - e^{-t/\beta_F \tau_F}\right)$$

$$V_{OUT} = 5 - (20)\left(1 - e^{-10^9 t}\right)$$



$$\boxed{t_{on} = 0.29ns}$$

c) The base charge just prior to the onset of saturation is defined as q_{B0}. In part b), you calculated the time at which the BJT transitions from forward active to saturation. Calculate q_{B0} using the forward active region charge control model. Calculate i_{B0}, which is the value of base current that biases the BJT right at the edge of saturation.

Hint: I_C is determined by the circuit, and $q_{B0} = q_F$ when $V_{CE} = V_{CE(sat)}$.

After $t \approx 0.288ns$, BJT is at edge of saturation

$$i_c(t = 0.288ns) = 100\,(100\mu)\left(1 - e^{(-10^9)(.288n)}\right) = 2.5mA$$

$$q_{B0} = q_F(t = 0.288ns) = \tau_F\, i_c(0.288ns) = (10p)(2.5m)$$

$$\boxed{q_{B0} = 0.025pC}$$

$$i_{B0} = \frac{i_c(t = 0.288ns)}{\beta_F} = \frac{2.5m}{100}$$

$$\boxed{i_{B0} = 25\mu A}$$

d) You now have enough information to calculate the value of q_S, the excess charge stored in the base at time t = ∞, assuming now that the BJT goes into saturation.

$$q_{total} = q_{B0} + q_s$$

$$q_s = q_{total} - q_{B0} = 3.41p - 0.025p$$

$$\boxed{q_s = 3.385pC}$$

e) Now assume the base current is switched back to 0 after q_S has been stored in the base. With $i_B = 0$, the differential equation for q_S becomes: $-i_{B0} = q_S/\tau_S + q_S/dt$. The final value of q_S from this equation is $-i_{B0}\tau_S$. In reality, a negative q_S is not allowed, and instead the device will enter the forward active region when $q_S = 0$. However, we use this final value to find the time when $q_S = 0$. Sketch the solution to the above differential equation with an initial value of q_S found in d), and the final value of $-i_{B0}\tau_S$. Calculate the value of τ_S. Now calculate the time at which $q_S = 0$. This is the time required to bring the BJT out of saturation, *after* which it will enter the forward active region and v_{out} begins to rise.

$-i_{B0} = q_s/\tau_s + q_s/dt$

$(1)\ \frac{dq_{sp}}{dt} + \frac{1}{\tau_s} q_{sp} = -i_{B_0}$, $(2)\ \frac{dq_{sc}}{dt} + \frac{1}{\tau_s} q_{sc} = 0$, q_{sp} ≡ particular solution, q_{sc} ≡ complementary solution

→ q_{sp} must be a constant since $-i_{B0}$ is constant. Therefore,

$$q_{sp} = -i_{B_0}\tau_s$$

→ Rearranging equation (2), we get

$$\frac{dq_{sc}/dt}{q_{sc}} = \frac{-1}{\tau_s}$$

$$\frac{d}{dt}\left[\ln q_{sc}\right] = \frac{-t}{\tau_s}$$

$$\ln q_{sc} = \frac{-t}{\tau_s} + C$$

$$q_{sc} = e^{C} e^{-t/\tau_s} = K e^{-t/\tau_s}$$

→ To calculate K, the initial condition is $q_s(t=0) = q_{s0}$ (from part d)

$$q_s(t) = q_{sp} + q_{sc} = -i_{B_0}\tau_s + K e^{-0/\tau_s} = q_{s0}$$

$$K = q_{s0} + i_{B_0}\tau_s$$

$$q_s(t) = -i_{B_0}\tau_s + (q_{s_0} + i_{B_0}\tau_s)\, e^{-t/\tau_s}$$

$$\boxed{q_s(t) = -i_{B_0}\tau_s\left(1 - e^{-t/\tau_s}\right) + q_{s_0} e^{-t/\tau_s}}$$

$i_{B_0} = 25\,\mu A$

$q_{s_0} = 3.385\,pC$

$\tau_s = \frac{\beta_F \tau_{BR} + (1+\beta_R)\tau_{BF}}{1+\beta_F+\beta_R}$

$\boxed{\tau_s = 45.14\,ns}$

For $q_s(t) = 0$ → $\boxed{t \approx 62.57\,ns}$