Problem 2.1: Use the circuit to the right for this problem. Ignore channel length modulation ($\lambda = 0$), devices $M_2$ and $M_3$ are identical. All devices are in saturation.

a) Find an expression for the DC small signal gain $A_v = \frac{v_{out}}{v_{in}}$.

b) If the DC bias $V_{GS1} - V_{TH1}$ is doubled, by what factor does the DC gain increase?

c) Consider only capacitances $C_{GS}$ and $C_{DB}$. Find the complete transfer function of the circuit.

d) Find expressions for the frequencies of any poles and zeros (i.e. $\omega_{p1, 2}, \omega_{z1, 2}$, etc.).

e) Assume that the DC bias $V_{GS1} - V_{TH1}$ is doubled. By what factors do the poles and zeros found in part d) increase or decrease? Sketch the transfer functions for nominal $V_{GS1} - V_{TH1}$ and $2(V_{GS1} - V_{TH1})$, labeling the differences between the plots and where they intersect/overlap.

Problem 2.2: BJT current mirrors have limited accuracy due to the finite base current. This problem addresses some of these limitations. For each part, assume the devices are in the forward active region, $\beta_f$ is the same for all devices, and ignore the Early effect ($V_A = \infty$).

a) For the circuit on the right, $A_E$ denotes the area of the emitter. $I_{OUT}$ should nominally be equal to $N I_S$, however due to finite base currents is is not. Find an expression for the large signal error current $I_{err} = N I_S - I_{OUT}$.

b) Find an expression for the large signal error current $I_{err} = N I_S - I_{OUT}$ for the circuit on the right with a transistor buffering the base currents.
**Problem 2.3:** Cascoding is used to increase output impedance. A cascode stage of two devices is shown on the right. For this problem, do not neglect $r_o$ or $r_{\pi}$. $V_B$ is a DC bias voltage and the two devices are matched with equal collector currents.

a) Find an expression for the small signal output impedance $R_{\text{OUT}}$ of the cascoded BJTs shown on the right.

b) Next consider a stack of three BJTs. Assuming $r_{\pi} \ll g_m r_o^2$, does the output impedance increase significantly if there are three stacked devices? Find an expression for the maximum $R_{\text{OUT}}$ that can be obtained as the number of stacked devices goes to infinity.

**Problem 2.4:** Calculate numerical values for the following parts assuming a temperature of 300K.

a) Calculate a numerical value for the open circuit rms noise voltage in a 1Hz bandwidth (called the “spot noise”) for a 50Ω resistor with only thermal noise. What is the open circuit rms noise voltage of this resistor in a 1MHz bandwidth?

b) Assuming the 50Ω resistor has a flicker noise component of $i_n^2 / \Delta f = 1.6 \cdot 10^{-19} / f$, sketch the noise spectral density $i_n^2 / \Delta f$ including flicker and thermal noise and calculate the noise corner frequency.

**Problem 2.5:** Use the circuit to the right for this problem.

a) Identify the noise sources in the circuit and draw the small signal circuit including these sources. Ignore flicker noise in all components. Find an expression for the mean-square noise spectral density $\overline{v_o^2} / \Delta f$ at the output.

b) Sketch the noise spectral density $\overline{v_o^2} / \Delta f$. Find an expression to approximate the equivalent noise bandwidth $\Delta f$ of a brickwall filter.

c) Find an expression for the total mean-square noise voltage $\overline{v_o^2}$ at the output. Hint: Your answer should not be in terms of any resistors.
Problem 2.6: Use the circuit to the right for this problem.

a) Calculate the DC bias current assuming the diode has an ideal on voltage of 0V. Draw the small signal model of the circuit and calculate the value of the diode small signal resistance at room temperature ($V_T = 25mV$).

b) Assuming thermal noise in the resistor and only shot noise in the diode, redraw the small signal circuit with the noise sources included.

c) Find an expression for the mean-square noise voltage spectral density $\tilde{v}_{n,o}^2/\Delta f$ at the output. Evaluate the total rms noise voltage $\tilde{v}_{n,o}$ at the output given a noise bandwidth of $\Delta f = 1MHz$. 

5V
10kΩ
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