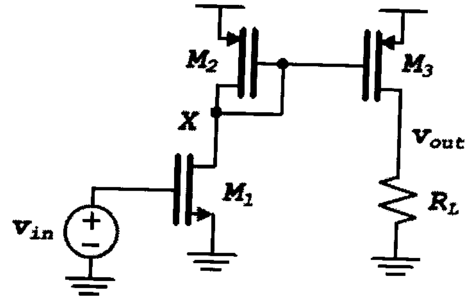
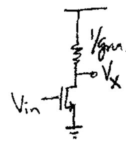
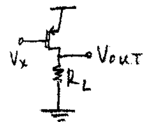
Problem 2.1: Use the circuit to the right for this problem. Ignore channel length modulation ($\lambda = 0$), devices M_2 and M_3 are identical. All devices are in saturation.



a) Find an expression for the DC small signal gain $A_v = v_{out}/v_{in}$.



$$A_v' = -g_{m_1}\left(\frac{1}{g_{m2}}\right)$$



$$A_v'' = -g_{m_3} R_L$$

$$A_v = A_v' A_v'' = \left(\frac{-g_{m_1}}{g_{m2}}\right)\left(-g_{m_3} R_L\right) \quad , \quad g_{m2} = g_{m3}$$

$$\boxed{A_v = g_m R_L}$$

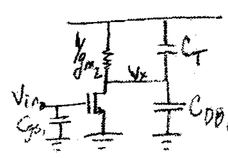
b) If the DC bias V_{IN} component of the input source v_{in} is increased so that $V_{GS1}-V_{TH1}$ is doubled, relate the new DC gain to the expression found in a).

$$g_m = \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{th}) \Rightarrow g_{m_{new}} = \mu_n C_{ox} \frac{W}{L}\left[2(V_{GS} - V_{th})\right]$$

$$g_{m_{new}} = 2 g_m$$

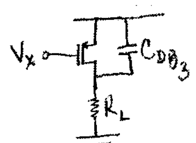
$$A_{v_{new}} = g_{m_{new}} R_L = \boxed{2 g_m R_L} \rightarrow \text{gain is doubled.}$$

c) Consider only capacitances C_{GS}, C_{DB}, and C_{SB}. Find the transfer function of the circuit.



$$C_T = C_{GS_2} + C_{DB_2} + C_{GS_3} = 2C_{GS_2} + C_{DB_2} \quad , \quad C_{GS_2} = C_{GS_3}$$

$$A_v' = -g_{m_1}\left[\left(\frac{1}{g_{m_2}}\right) \Big\| \left(\frac{1}{s(C_T + C_{DB_1})}\right)\right] = -g_{m_1}\left[\frac{1}{g_{m_2} + s(C_T + C_{DB_1})}\right]$$



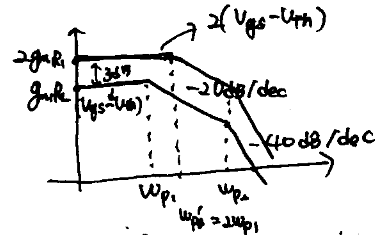
$$A_v'' = -g_{m_3}\left[R_L \Big\| \frac{1}{sC_{DB_3}}\right] = -g_{m_3}\left(\frac{R_L}{1 + sR_L C_{DB_3}}\right)$$

$$A_v = A_v' \cdot A_v'' = \boxed{\frac{g_{m_1} g_{m_3} R_L}{\left[g_{m_2} + s(C_T + C_{DB_1})\right]\left(1 + sR_L C_{DB_3}\right)}}$$

d) Find expressions for the frequencies of any poles and zeros (i.e. ω_{p1}, ω_{z1}, etc.).

$$P_1 = -\frac{g_{m2}}{C_T + C_{DB_1}} = \boxed{\frac{-g_{m2}}{2C_{GS_2} + C_{DB_2} + C_{DB_1}}} \qquad \boxed{P_2 = \frac{-1}{R_L C_{DB_3}}}$$

e) Assuming that the DC bias V_{GS1}–V_{TH1} is doubled, relate the new pole and zero frequencies to the expressions found in d).



$$I_{D_1} = I_{D_2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS_1} - V_{th_1}\right)^2$$

$$I_{D_2 new} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2\left(V_{GS_1} - V_{th_1}\right)\right]^2 = 4I_D$$

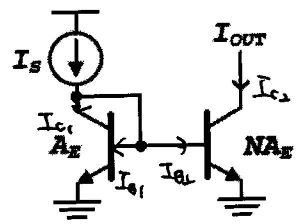
$$g_{m_2} = \frac{2I_{D_2}}{V_{GS_2} - V_{th_2}} \longrightarrow g_{m_2 new} = \frac{2I_{D_2 new}}{2(V_{GS_2} - V_{th_2})} = \frac{8I_{D_2}}{2(V_{GS_2} - V_{th_2})} = \frac{4I_{D_2}}{V_{GS_2} - V_{th_2}}$$

$$g_{m_2 new} = 2g_{m_2}$$

thus $\boxed{P_{1\,new} = 2P_1}$ P_1 is doubled P_2 stays the same

Problem 2.2: BJT current mirrors have limited accuracy due to the finite base current. This problem addresses some of these limitations. For each part, assume the devices are in the forward active region, β is the same for all devices, and ignore the Early effect.

a) For the circuit on the right, A_E denotes the area of the emitter. I_{OUT} should nominally be equal to NI_S. Find an expression for the large signal error current $I_E = NI_S - I_{OUT}$.



$$V_{BE_1} = V_{BE_2} \quad I_{B_1} = \frac{I_{B_2}}{N} \quad I_{C_1} = \frac{I_{C_2}}{N} \quad I_{C_2} = I_{OUT}$$

$$I_S = I_{C_1} + I_{B_1} + I_{B_2} = \frac{I_{C_2}}{N} + \frac{I_{B_2}}{N} + I_{B_2} = \frac{I_{C_2}}{N} + I_{B_2}\left[\frac{1}{N}+1\right]$$

$$= \frac{I_{C_2}}{N} + I_{B_2}\left(\frac{N+1}{N}\right) = \frac{I_{C_2}}{N} + \frac{I_{C_2}}{\beta}\left[\frac{N+1}{N}\right]$$

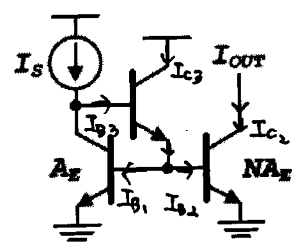
$$= I_{C_2}\left[\frac{1}{N} + \frac{N+1}{\beta N}\right] = I_{OUT}\left[\frac{\beta+N+1}{\beta N}\right] \Longrightarrow I_{OUT} = I_S\left[\frac{\beta N}{\beta+N+1}\right]$$

$$I_E = NI_S - I_{OUT} = NI_S - I_S\left[\frac{\beta N}{\beta+N+1}\right]$$

$$= I_S\left[N - \frac{\beta N}{\beta+N+1}\right]$$

$$\boxed{I_E = I_S\left[\frac{(1+N)(N)}{\beta+N+1}\right]}$$

b) Find an expression for the large signal error current $I_E = NI_S - I_{OUT}$ for the circuit on the right with a transistor buffering the base currents.



$$I_{C_2} = I_{OUT} \quad I_{B_3} = \frac{I_{B_1} + I_{B_2}}{\beta+1} \quad I_{B_1} = \frac{I_{B_2}}{N} \quad I_{C_1} = \frac{I_{C_2}}{N}$$

$$I_S = I_{C_1} + I_{B_3} = I_{C_1} + \frac{I_{B_1} + I_{B_2}}{\beta+1} = I_{C_1} + \left[\frac{N+1}{N}\right]\left[\frac{1}{\beta+1}\right]I_{B_2}$$

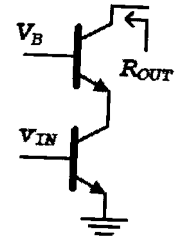
$$= \frac{I_{C_2}}{N} + \frac{I_{C_2}}{\beta}\left[\frac{N+1}{N(\beta+1)}\right] = I_{OUT}\left[\frac{1}{N} + \frac{N+1}{\beta N(\beta+1)}\right]$$

$$I_{OUT} = I_S\left[\frac{\beta(\beta+1)N}{\beta(\beta+1)+(N+1)}\right]$$

$$I_E = NI_S - I_{OUT} = I_S\left[N - \frac{\beta(\beta+1)(N)}{\beta(\beta+1)+(N+1)}\right]$$

$$\boxed{I_E = I_S\left[\frac{(N+1)(N)}{\beta(\beta+1)+(N+1)}\right]}$$

Problem 2.3: Cascoding is used to increase output impedance. A cascode stage of two devices is shown on the right. For this problem, do not neglect r_o or r_π. V_B is a DC bias voltage and the two devices are matched with equal collector currents.

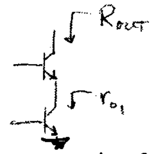


a) Find an expression for the small signal output impedance R_{OUT} of the cascoded BJTs shown on the right.

In general,

$$R_o = (r_\pi \| R) + \left[1 + g_m (r_\pi \| R)\right] r_o$$

For our case,

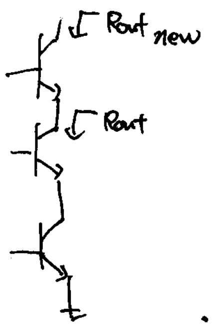


$$\boxed{R_{OUT} = (r_{\pi_2} \| r_{o_1}) + \left[1 + g_{m_2}(r_{\pi_2} \| r_{o_1})\right] r_{o_2}}$$

b) Next consider a stack of three BJTs. Assuming $r_\pi \ll g_m r_o^2$, does the output impedance increase significantly if there are three stacked devices? Find an expression for the maximum R_{OUT} that can be obtained as the number of stacked devices goes to infinity.

$$R_{OUT} \approx \left[1 + g_{m_2}(r_{\pi_2} \| r_{o_1})\right] r_{o_2} \approx g_{m_2}(r_{\pi_2} \| r_{o_1}) r_{o_2}$$

$$R_{OUT} \approx g_{m_2} r_{\pi_2} r_{o_2}$$



$$R_{OUT_{New}} = (r_{\pi_3} \| R_{OUT}) + \left[1 + g_{m_3}(r_{\pi_3} \| R_{OUT})\right] r_{o_3}$$

$$R_{OUT_{New}} \approx g_{m_3}(r_{\pi_3} \| g_{m_2} r_{\pi_2} r_{o_2}) r_{o_3}$$

$$R_{OUT_{new}} \approx g_{m_3} r_{\pi_3} r_{o_3}$$

Thus, $\boxed{R_{OUT} \approx R_{OUT_{new}}}$

$\therefore$ as $n \to \infty$, $R_{out} \approx g_{m_3} r_{\pi_3} r_{o_3} = \beta r_o$

Problem 2.4: Calculate numerical values for the following parts assuming a temperature of 300K.

a) Calculate a numerical value for the open circuit rms spot noise voltage (1Hz bandwidth) for a 50Ω resistor with only thermal noise. What is the total open circuit rms noise voltage of this resistor in a 1MHz bandwidth?

$$\overline{V_R} = \sqrt{4kTR\Delta f} = \sqrt{4(1.38 \times 10^{-23})(300)(50)}\sqrt{\Delta f}$$

$$\frac{\overline{V_R}}{\sqrt{\Delta f}} = \boxed{0.91\ nV/\sqrt{Hz}}$$

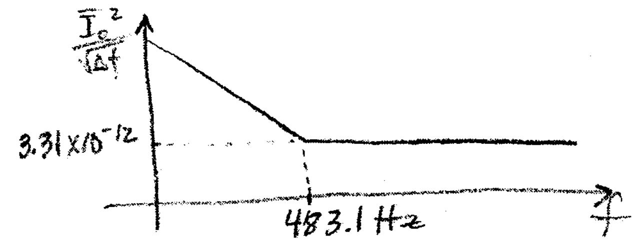
$$\overline{V_R} = \frac{\overline{V_R}}{\sqrt{\Delta f}}\sqrt{\Delta f} = (0.91\ n)\sqrt{1 \times 10^6} = \boxed{0.91\ \mu V}$$

b) Assuming the 50Ω resistor has a flicker noise component of $\overline{i_n^2}/\Delta f = 1.6\cdot10^{-19}/f$, sketch the noise spectral density $\overline{i_n^2}/\Delta f$ including flicker and thermal noise and calculate the noise corner frequency.

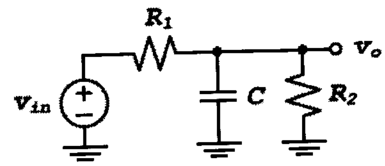
$$\frac{\overline{i_n^2}}{\Delta f} = \frac{\overline{i_R^2}}{\Delta f}$$

$$1.6\times10^{-19}/f = 3.31\times10^{-22}$$

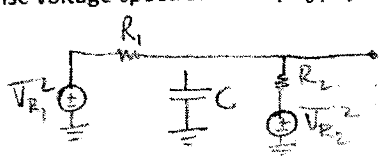
$$f = \frac{1.6\times10^{-19}}{3.31\times10^{-22}} = \boxed{483.1\ Hz}$$



Problem 2.5: Use the circuit to the right for this problem.



a) Identify the noise sources in the circuit and draw the small signal circuit including these sources. Ignore flicker noise in all components. Find an expression for the mean-square noise voltage spectral density $\overline{v_o^2}/\Delta f$ at the output.



$$V_o = \frac{R_2 // \frac{1}{sC}}{R_1 + R_2 // \frac{1}{sC}} V_{R_1} + \frac{R_1 // \frac{1}{sC}}{R_2 + R_1 // \frac{1}{sC}} V_{R_2}$$

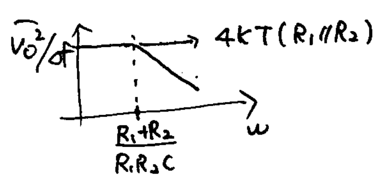
$$\overline{V_{R_1}^2} = 4kR_1T\Delta f, \quad \overline{V_{R_2}^2} = 4kR_2T\Delta f$$

$$\overline{V_o^2} = \left|\frac{R_2}{(R_1+R_2)+sR_1R_2C}\right|^2 4kR_1T\Delta f + \left|\frac{R_1}{(R_1+R_2)+sR_1R_2C}\right|^2 4kR_2T\Delta f$$

$$\frac{\overline{V_o^2}}{\Delta f} = \frac{4kTR_1R_2(R_1+R_2)}{(R_1+R_2)^2+R_1^2R_2^2C^2\omega^2} = \frac{4kTR_1R_2}{R_1+R_2}\cdot\frac{1}{1+\frac{R_1^2R_2^2C^2}{(R_1+R_2)^2}\omega^2}$$

$$\omega_{p1} = \frac{R_1+R_2}{R_1R_2C}$$

b) Sketch the noise spectral density $\overline{v_o^2}/\Delta f$. Find an expression to approximate the equivalent noise bandwidth Δf of a brickwall filter.



$$\overline{V_{total}^2} = \int_0^\infty \frac{\overline{V_o^2}}{df}\,df = \frac{4kTR_1R_2}{R_1+R_2}\int_0^\infty \frac{1}{1+\frac{R_1^2R_2^2C^2}{(R_1+R_2)^2}4\pi^2f^2}\,df$$

$$= \frac{4kTR_1R_2}{R_1+R_2}\cdot\frac{R_1+R_2}{R_1R_2C\,2\pi}\cdot\left[\tan^{-1}\left(\frac{2\pi f R_1R_2C}{R_1+R_2}\right)\right]_0^\infty$$

$$= \frac{kT}{C}$$

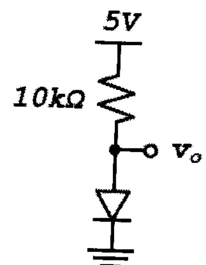
$$\frac{\overline{V_{peak}^2}}{df} = \frac{4kTR_1R_2}{R_1+R_2} = 4kT(R_1//R_2),\ \Delta f = \frac{\overline{V_{total}^2}}{\frac{\overline{V_{peak}^2}}{df}} = \frac{\frac{kT}{C}}{\frac{4kTR_1R_2}{R_1+R_2}} = \frac{1}{4(R_1//R_2)\cdot C}$$

c) Find an expression for the total mean-square noise voltage $\bar{v}_o^2$ at the output. Hint: Your answer should not be in terms of any resistors.

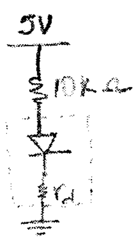
$$\overline{V_o^2} = \left(\frac{\overline{V_o^2}}{\Delta f}\right)\Delta f = \left[4kT(R_1||R_2)\right]\left[\frac{1}{4(R_1||R_2)C}\right]$$

$$\boxed{\overline{V_o^2} = \frac{kT}{C}}$$

Problem 2.6: Use the circuit to the right for this problem.



a) Calculate the DC bias current assuming the diode has an ideal on voltage of 0V. Draw the small signal model of the circuit and calculate the value of the diode small signal resistance at room temperature (V_T = 25mV).

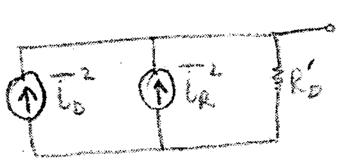


$$r_d = \frac{nV_T}{I_D}, \quad n \approx 1$$

$$I_D = \frac{5}{10k} = \boxed{0.5mA}$$

$$r_d = \frac{26m}{0.5m} = \boxed{52\,\Omega}$$

b) Assuming thermal noise in the resistor and only shot noise in the diode, redraw the small signal circuit with the noise sources included.



$$\overline{i_D^2} = 2qI_D\Delta f$$

$$\overline{i_R^2} = 4kT\frac{1}{R}\Delta f \qquad R = 10k\Omega$$

$$R_D' = r_d || R \approx 50\Omega$$

c) Find an expression for the mean-square noise voltage spectral density $\bar{v}_{no}^2/\Delta f$ at the output. Evaluate the total rms noise voltage $\bar{v}_{no}$ at the output given a noise bandwidth of $\Delta f = 1MHz$.

$$\frac{\overline{V_o^2}}{\Delta f} = \left(\overline{i_D^2} + \overline{i_R^2}\right)R_D'^2 = \left(2qI_DR'^2 + 4kTR\right)$$

$$\frac{\overline{V_o^2}}{\Delta f} = \boxed{2qI_DR_D'^2 + \frac{4kTR_D'^2}{R}}$$

$$\overline{V_o^2} = \left[2(1.6\times10^{-19})(0.5m)(50)^2 + 4(1.38\times10^{-23})(300)\frac{(50^2)}{10k}\right](1M)$$

$$\boxed{V_o \approx 0.64\mu V}$$