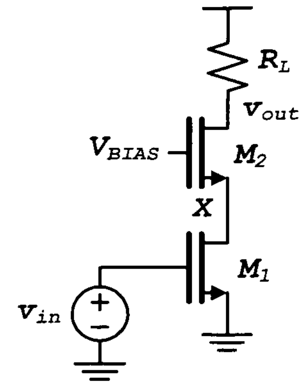
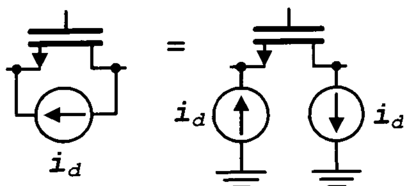
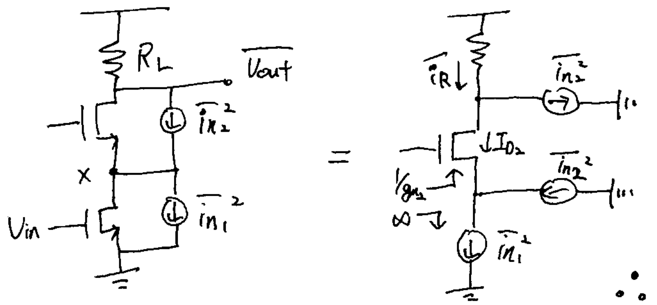
Problem 3.1: Use the circuit to the right for this problem. Ignore channel length modulation ($\lambda = 0$). All devices are in saturation. Consider only drain thermal noise in the FETs (no thermal noise in R_L). Find an expression for the input referred noise voltage neglecting induced gate noise. How does M_2 affect the noise contributed by the circuit?
Hint: the following circuit transformation may be useful.







. KCL

$$\overline{i_R} - \overline{i_{n_2}} = I_{D_2} = \overline{i_{n_1}} - \overline{i_{n_2}}$$

$$\therefore \overline{i_R} = \overline{i_{n_1}}$$

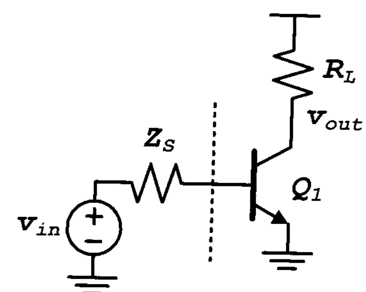
$$\therefore \overline{V_{out_n}^2} = \overline{i_{n_1}^2} R_L^2 = 4kT\gamma g_{m_1} R_L^2 \Delta f$$

Input-referred noise:

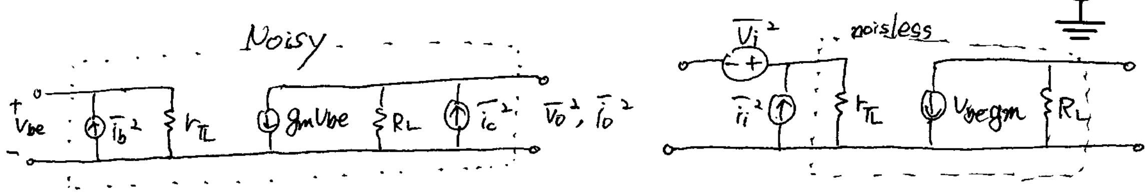
$$\overline{V_{in_n}^2} = \frac{\overline{V_{out_n}^2}}{A_v^2} = \frac{4kT\gamma g_{m_1} R_L^2}{g_{m_1}^2 R_L^2}\Delta f$$

$$\therefore \frac{\overline{V_{in_n}^2}}{\Delta f} = 4kT\gamma\frac{1}{g_{m_1}}$$ $\Rightarrow$ M_2 does not affect the noise.

Problem 3.2: Use the circuit to the right for this problem. Assume the BJT is in the forward active region, ignore r_o and r_b. Consider only shot noise in the collector and base (no thermal noise in R_L). Assume β is constant with frequency.



a) Derive expressions for the input-referred short-circuit noise voltage and open-circuit noise current (not including Z_S).



To get $\overline{V_i^2}$, short both inputs (noisy and noisless)

$$\overline{V_o^2} = [(g_m V_{be})^2 + \overline{i_c^2}] R_L^2, \quad V_{be} = 0$$

$$\overline{V_o^2} = \overline{i_c^2} R_L^2$$

$$\overline{V_i^2} = \frac{\overline{V_o^2}}{A_v^2} = \frac{\overline{i_c^2} R_L^2}{g_m^2 R_L^2} = \frac{\overline{i_c^2}}{g_m^2} = \frac{2qI_c\Delta f}{g_m^2}$$

$$\boxed{\frac{\overline{V_i^2}}{\Delta f} = 2qI_c\frac{1}{g_m^2}}$$

To get $\overline{i_i^2}$, open both inputs

$$\begin{cases} \overline{i_o^2} = (g_m V_{be})^2 + \overline{i_c^2} \\ \overline{i_o^2} = g_m^2 \overline{i_b^2} r_\pi^2 + \overline{i_c^2} \end{cases}$$

$$\cdot\ \overline{i_o^2} = (g_m V_{be})^2 = (g_m \overline{i_i} r_\pi)^2$$

$$\therefore g_m^2 \overline{i_i^2} r_\pi^2 = g_m^2 \overline{i_b^2} r_\pi^2 + \overline{i_c^2}$$

$$\overline{i_i^2} = \overline{i_b^2} + \frac{\overline{i_c^2}}{g_m^2 r_\pi^2} = \overline{i_b^2} + \frac{\overline{i_c^2}}{\beta^2}$$

$$\overline{i_i^2} = 2q\left(I_B + \frac{I_c}{\beta^2}\right)\Delta f = 2q\left(I_B + \frac{I_B}{\beta}\right)\Delta f$$

$$\boxed{\frac{\overline{i_i^2}}{\Delta f} = 2qI_B\frac{\beta+1}{\beta}}$$

b) Derive an expression for the correlation admittance Y_c.

$$i_c = \frac{\overline{i_c}}{g_m r_\pi}, \quad V_i = \frac{\overline{i_c}}{g_m}$$

$$Y_c = \frac{i_c}{V_i} = \boxed{\frac{1}{r_\pi}}$$

c) Derive expressions for the 4 noise parameters Gc, Bc, Rn, and Gu.

$$R_n = \frac{\overline{V_i^2}}{4KT\Delta f} = \frac{2qI_c\frac{1}{g_m^2}\Delta f}{4KT\Delta f} = \frac{1}{2}\left(\frac{q}{KT}\right)I_c\frac{1}{g_m^2} = \frac{1}{2}\frac{I_c}{V_T}\cdot\frac{1}{g_m^2} = \frac{1}{2g_m}$$

$$Y_c = G_c + jB_c \Rightarrow G_c = \frac{1}{r_\pi}, \quad B_c = 0$$

$$G_u = \frac{\overline{i_b^2}}{4KT\Delta f} = \frac{2qI_B\Delta f}{4KT\Delta f} = \frac{1}{2}\left(\frac{q}{KT}\right)I_B = \frac{g_m}{2\beta}$$

d) Find the source impedance resulting in minimum noise factor and evaluate is assuming $I_{C1} = 1mA$ and $\beta_F = 100$.

$$B_{S,opt} = -B_c = 0$$

$$G_{S,opt} = \sqrt{\frac{G_u}{R_n} + G_c^2} = \sqrt{\frac{I_B}{I_c}g_m^2 + \frac{1}{r_\pi^2}}$$

$$G_{S,opt} = \sqrt{\frac{g_m}{r_\pi} + \frac{1}{r_\pi^2}} = \sqrt{\frac{I_c^2}{V_T^2}\cdot\frac{1}{\beta} + \frac{1}{\beta^2}\cdot\frac{I_c^2}{V_T^2}}$$

$$\doteq 0.004$$

Problem 3.3: Suppose you have a choice between two amplifiers, both having 10nV/√Hz input noise voltage density; however, amplifier A has 50fA/√Hz input noise current density and amplifier B has 100fA/√Hz.

a) What is the optimum source resistance (resulting in lowest noise factor) for each amplifier? Assume the input noise sources are uncorrelated.

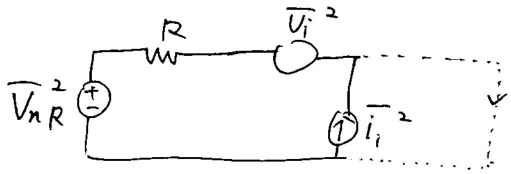
$$Y_s = G_{S,opt} = \sqrt{\frac{G_u}{R_n} + G_c^2}, \quad G_c = 0$$

$$G_u = \frac{\overline{i_i^2}}{4KT\Delta f}, \quad R_n = \frac{\overline{V_i^2}}{4KT\Delta f}$$

$$Y_s = \sqrt{\frac{\overline{i_i^2}}{\overline{V_i^2}}} \Rightarrow R_S = \frac{\overline{V_i}}{\overline{i_i}}$$

$$R_{S_A} = \frac{10n}{50f} = \boxed{200\,k\Omega}, \quad R_{S_B} = \frac{10n}{100f} = \boxed{100\,k\Omega}$$

b) If the source resistance is 100kΩ, which amplifier should you use for lowest noise?



$$F = \frac{\dfrac{\overline{V_{nR}^2} + \overline{V_i^2}}{R^2} + \overline{i_i^2}}{\overline{V_{nR}^2}/R^2} = \frac{\overline{V_{nR}^2} + \overline{V_i^2} + \overline{i_i^2}R^2}{\overline{V_{nR}^2}}$$

$$F = 1 + \frac{\overline{V_i^2}}{\overline{V_{nR}^2}} + \frac{\overline{i_i^2}R^2}{\overline{V_{nR}^2}} = 1 + \frac{\overline{V_i^2}}{4KTR\Delta f} + \frac{\overline{i_i^2}R}{4KT\Delta f}$$

→ For Amp A,

$R_{SA} = 200\,k\Omega$

$$F = 1 + \frac{(10nV)^2}{4\cdot 1.38\cdot 10^{-23}\cdot 300\cdot 200k} + \frac{(50f)^2\cdot(200k)}{4\cdot 1.38\times 10^{-23}\cdot 300}$$

$= 1.076$

→ For Amp B,

$$F = 1 + \frac{(10nV)^2}{4\cdot 1.38\cdot 10^{-23}\cdot 300\cdot 100k} + \frac{(100f)^2\cdot 100k}{4\cdot 1.38\cdot 10^{-23}\cdot 300}$$

$= 1.121$

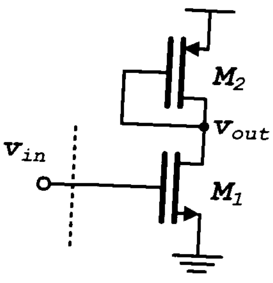
Therefore Amplifier A is the better option.

c) For your choice in part b), what is the noise factor?

From part b), $F = 1.076$

Problem 3.4: Find expressions for the mean-square input referred short-circuit noise voltage and open-circuit noise current for the following circuits. You may neglect r_o and g_{mb}, and r_b. Include only drain noise in FET's, thermal noise in R's, and base and collector shot noise in BJT's.

a)



$$\overline{V_{out}}^2 = 4KT(\gamma_n g_{m1} + \gamma_p g_{m2})\left(\frac{1}{g_{m2}}\right)^2 \Delta f$$

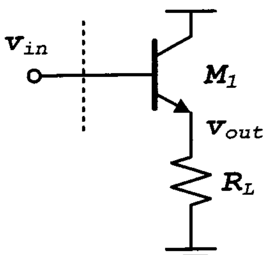
$$\overline{V_{in}}^2 = \frac{\overline{V_{out}}^2}{A_v^2}, \quad A_v = -\frac{g_{m1}}{g_{m2}}$$

$$\frac{\overline{V_{in}}^2}{\Delta f} = 4KT(\gamma_n g_{m1} + \gamma_p g_{m2})\frac{1}{g_{m1}^2}$$

$$Z_{in} = \frac{1}{j\omega C_{gs}}$$

$$\frac{\overline{I_{in}}^2}{\Delta f} = \frac{\overline{V_{in}}^2}{Z_{in}^2}\frac{1}{\Delta f} = \omega^2 C_{gs}^2 \cdot 4KT(\gamma_n g_{m1} + \gamma_p g_{m2})\frac{1}{g_{m1}^2}$$

b)



$$V_{out}\big|_{V_{in}=0} = (i_b + i_c + i_{RL})\cdot\left(\frac{1}{g_m} \,\|\, R_L \,\|\, \frac{1}{j\omega C_\pi}\right)$$

$$\overline{V_{out}}^2 = \left(\overline{i_b}^2 + \overline{i_c}^2 + \overline{i_{RL}}^2\right)\left(\frac{R_L}{1 + g_m R_L + j\omega C_\pi R_L}\right)$$

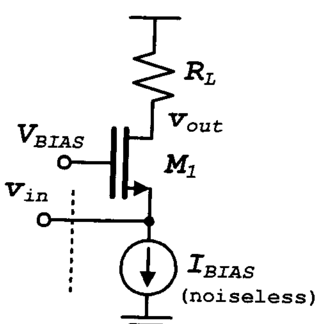
$A_v = 1$ $\quad V_{out} \approx V_{in} \Rightarrow \overline{V_{in}}^2 = \overline{V_{out}}^2$

$$\frac{\overline{V_{in}}^2}{\Delta f} = \left(2qI_B + 2qI_C + \frac{4KT}{R_L}\right)\cdot\left(\frac{R_L}{1 + g_m R_L + j\omega R_L C_\pi}\right)^2$$

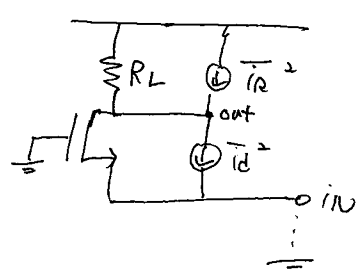
$$= \frac{2qR_L^2 I_C\left(1+\frac{1}{\beta}\right) + 4KTR_L}{(1+g_m R_L)^2 + \omega^2 R_L^2 C_\pi^2}$$

$$\frac{\overline{I_{in}}^2}{\Delta f} = \left|R_L + r_\pi \| \frac{1}{j\omega C_\pi} + R_L g_m \cdot r_\pi \| \frac{1}{j\omega C_\pi}\right|^2 \cdot \frac{2qR_L^2 I_C\left(1+\frac{1}{\beta}\right) + 4KTR_L}{(1+g_m R_L)^2 + \omega^2 R_L^2 C_\pi^2}$$

c)



$\Rightarrow$



$$\overline{V_{out}}^2 = \left[4KT\gamma g_m + 4KT\frac{1}{R_L}\right]R_L^2$$

$$A_v^2 = (g_m + g_{mb})^2 R_L^2 \approx g_m^2 R_L^2$$

$$\overline{V_{in}}^2 = \frac{\overline{V_{out}}^2}{A_v^2} = \left(4KT\gamma g_m + 4KT\frac{1}{R_L}\right)\frac{\Delta f}{g_m^2}$$

$$\Rightarrow \frac{\overline{V_{in}}^2}{\Delta f} = 4KT\left(\gamma g_m + \frac{1}{R_L}\right)\left(\frac{1}{g_m^2}\right)$$

$$\frac{\overline{I_{in}}^2}{\Delta f} = \left(\omega^2 C_{gs}^2 + \frac{1}{g_m^2}\right) 4KT\left(\gamma g_m + \frac{1}{R_L}\right)\frac{1}{g_m^2}$$