Problem 3.1: Use the circuit to the right for this problem. Ignore channel length modulation ($\lambda = 0$). All devices are in saturation. Consider only drain thermal noise in the FETs (no thermal noise in $R_L$). Find an expression for the input referred noise voltage neglecting induced gate noise. How does $M_2$ affect the noise contributed by the circuit? Hint: the following circuit transformation may be useful.

\[ V_{in} = \frac{V_{out}}{A^2} = \frac{4kTg_{m1}R_L}{g_{m1}^2R_L^2} \Delta f \]

Input referred noise:
\[ \overline{V_{in}^2} = \frac{\overline{V_{out}^2}}{A^2} = \frac{4kTg_{m1}R_L}{g_{m1}^2R_L^2} \Delta f \]
\[ \frac{\overline{V_{in}^2}}{\Delta f} = 4kT \frac{1}{g_{m1}} \Rightarrow M_2 \text{ does not affect the noise.} \]

Problem 3.2: Use the circuit to the right for this problem. Assume the BJT is in the forward active region, ignore $r_o$ and $r_b$. Consider only shot noise in the collector and base (no thermal noise in $R_L$). Assume $\beta$ is constant with frequency.

a) Derive expressions for the input-referred short-circuit noise voltage and open-circuit noise current (not including $Z_s$).

To get $\overline{V_i^2}$, short both inputs (noisy and noiseless):
\[ \overline{V_0} = [\overline{(g_mV_{be})^2 + \overline{v_c}^2}] R_L, \quad \overline{V_{be}} = 0 \]
\[ \overline{V_0} = \overline{V_c} R_L \]
\[ \frac{\overline{V_c}}{\overline{V_i}} = \frac{\overline{V_c} R_L}{g_m R_L^2} = \frac{\overline{i_c}}{g_m^2} \frac{1}{g_m^2} = 2qI_c \Delta f \]
\[ \frac{\overline{V_i^2}}{\Delta f} = 2qI_c \frac{1}{g_m^2} \]

To get $\overline{i_i}$, open both inputs:
\[ \overline{i_0} = (\overline{g_mV_{be}})^2 + \overline{i_c}^2 \]
\[ \overline{i_0} = g_m \overline{V_{be}}^2 + \overline{i_c}^2 \]
\[ \overline{i_0} = (\overline{g_mV_{be}}) = (g_m \overline{V_{be}})^2 \]
\[ \overline{V_{be}} = \overline{V_c} R_L \]
\[ \overline{i_i} = \overline{i_b} + \overline{i_c} = \overline{i_b} + \overline{i_c} \]
\[ \overline{i_i} = 2q(I_b + \frac{\overline{V_c}}{\beta}) \Delta f = 2q(I_b + \frac{V_c}{\beta}) \Delta f \]
\[ \frac{\overline{i_i^2}}{\Delta f} = 2qI_b\left(1 + \frac{1}{\beta}\right) \]
b) Derive an expression for the correlation admittance \( Y_c \).

\[
i_c = \frac{\overline{I_c}}{g_m R_n}, \quad V_i = \frac{\overline{I_c}}{g_m}
\]

\[
Y_c = \frac{i_c}{V_i} = \frac{1}{\overline{I_c}}
\]

c) Derive expressions for the 4 noise parameters \( G_c, B_c, R_n, \) and \( G_u \).

\[
R_n = \frac{V_i^2}{4kT \Delta f} = \frac{2q^2 I_c \Delta f}{4kT \Delta f} = \frac{1}{2} \left( \frac{q^2}{kT} \right) I_c \frac{1}{g_m^2} = \frac{1}{2} \frac{I_c}{v_r} = \frac{1}{2g_m}
\]

\[
Y_c = G_c + jB_c \quad \Rightarrow \quad G_c = \frac{1}{\overline{I_c}}, \quad B_c = 0
\]

\[
G_u = \frac{V_i^2}{4kT \Delta f} = \frac{2q^2 I_b \Delta f}{4kT \Delta f} = \frac{1}{2} \left( \frac{q^2}{kT} \right) I_b = \frac{g_m}{2\beta}
\]

d) Find the source impedance resulting in minimum noise factor and evaluate is assuming \( I_{c1} = 1mA \) and \( \beta_r = 100 \).

\[
B_{S_{opt}} = -B_c = 0
\]

\[
G_{S_{opt}} = \sqrt{\frac{G_u}{R_n} + G_c^2} = \sqrt{\frac{I_b}{I_c} \frac{g_m^2}{g_m^2} + \frac{1}{\overline{I_c}^2}}
\]

\[
G_{S_{opt}} = \sqrt{\frac{g_m}{\overline{I_c} \overline{I_c}} + \frac{1}{\overline{I_c}^2}} = \sqrt{\frac{I_b^2}{v_r^2} \frac{1}{\beta} + \frac{1}{\overline{I_c}^2} \frac{1}{\overline{I_c}^2}}
\]

\[
= 0.0044
\]

Problem 3.3: Suppose you have a choice between two amplifiers, both having 10nV/\sqrt{Hz} input noise voltage density; however, amplifier A has 50fA/\sqrt{Hz} input noise current density and amplifier B has 100fA/\sqrt{Hz}.

a) What is the optimum source resistance (resulting in lowest noise factor) for each amplifier? Assume the input noise sources are uncorrelated.

\[
Y_s = G_{S_{opt}} = \sqrt{\frac{G_u}{R_n} + G_c^2}, \quad G_c = 0
\]

\[
G_u = \frac{\overline{I_i}^2}{4kT \Delta f}, \quad R_n = \frac{\overline{V_i}^2}{4kT \Delta f}
\]

\[
Y_s = \frac{\overline{I_i}}{\overline{V_i}}, \quad \Rightarrow \quad R_S = \frac{\overline{V_i}}{\overline{I_i}}
\]

\[
R_{S_A} = \frac{10n}{50f} = \frac{100}{50} \approx 200 \Omega, \quad R_{S_B} = \frac{10n}{100f} = 100 \Omega
\]
b) If the source resistance is 100kΩ, which amplifier should you use for lowest noise?

\[ F = \frac{\overline{V_nR}^2 + \overline{i_n}^2}{\overline{V_nR}^2} = \frac{\overline{V_nR}^2 + \overline{V_i}^2 + \overline{i_i}^2 R^2}{\overline{V_nR}^2} = 1 + \frac{\overline{V_i}^2}{4kT Rof} + \frac{\overline{i_i}^2 R}{4kT of} \]

For Amp A,
\[ R_{SA} = 200 \text{k}\Omega \]
\[ F = 1 + \frac{(10\text{mV})^2}{4 \times 1.38 \times 10^{-3} \times 300 \times 200} + \frac{(50 \text{f})^2 \times 100k}{4 \times 1.38 \times 10^{-3} \times 300} \]
\[ = 1.076 \]

For Amp B,
\[ F = 1 + \frac{(10\text{mV})^2}{4 \times 1.38 \times 10^{-3} \times 300 \times 100} + \frac{(100\text{f})^2 \times 100k}{4 \times 1.38 \times 10^{-3} \times 300} \]
\[ = 1.121 \]

Therefore, Amplifier A is the better option.

c) For your choice in part b), what is the noise factor?

From part b), \[ F = 1.076 \]
Problem 3.4: Find expressions for the mean-square input referred short-circuit noise voltage and open-circuit noise current for the following circuits. You may neglect $r_o$ and $g_{mbr}$ and $r_b$. Include only drain noise in FET’s, thermal noise in R’s, and base and collector shot noise in BJT’s.

a)\[
\overline{V_{out}}^2 = 4kT \left( \frac{1}{g_{m1} + g_{m2}} \right) \left( \frac{1}{g_{m1}} \right)^2 \Delta f
\]
\[
\overline{V_{in}}^2 = \frac{\overline{V_{out}}^2}{A_v^2}, \quad A_v = -\frac{g_{m1}}{g_{m2}}
\]
\[
\frac{\overline{V_{in}}^2}{\Delta f} = 4kT \left( \frac{1}{g_{m1} + g_{m2} + j\omega C_g} \right) \left( \frac{1}{g_{m1}} \right)^2
\]
\[
\frac{\overline{I_{in}}^2}{\Delta f} = \frac{P_{in}}{\overline{V_{in}}^2} \Delta f = \frac{1}{2} \left( \frac{g_{m1}^2 + g_{m2}^2}{g_{m1}} \right) 4kT \left( \frac{1}{g_{m1}} \right)^2
\]

b)\[
\overline{V_{out}} \mid \overline{V_{in}} = \left( i_b + i_c + i_{RL} \right) \left( \frac{1}{g_{m1} + 1/R_L} \right) \left( \frac{1}{j\omega C_{RL}} \right)
\]
\[
\overline{V_{out}}^2 = \left( \overline{i_b}^2 + \overline{i_c}^2 + \overline{i_{RL}}^2 \right) \left( \frac{R_{L}}{1 + g_{m1}R_L + j\omega C_{RL}} \right)
\]
\[
A_v = 1, \quad \overline{V_{out}} \propto \overline{V_{in}} \Rightarrow \overline{V_{in}}^2 = \overline{V_{out}}^2
\]
\[
\frac{\overline{V_{in}}^2}{\Delta f} = \left( 2 \frac{g_{m1} R_L}{1 + g_{m1} R_L + j\omega C_{RL}} \right) \left( \frac{R_{L}}{1 + g_{m1} R_L + j\omega C_{RL}} \right)
\]
\[
= 2 \frac{g_{m1} R_L^2 I_C \left( H_L \right)}{\left( 1 + g_{m1} R_L \right)^2 + \omega^2 R_L^2 C_{RL}^2}
\]
\[
\frac{\overline{I_{in}}^2}{\Delta f} = \frac{1}{R_L + \frac{1}{j\omega C_{RL}}} + \frac{R_L g_{m1} R_{RL} \frac{1}{j\omega C_{RL}}}{\left( 1 + g_{m1} R_L \right)^2 + \omega^2 R_L^2 C_{RL}^2}
\]

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c)\[
\overline{V_{out}}^2 = \left[ 4kT \frac{g_{m1}}{R_L} + 4kT \frac{1}{R_L} \right] R_L^2
\]
\[
A_v = \frac{g_{m1} + g_{m2} R_L^2}{g_{m1} R_L} \approx g_{m2} R_L^2
\]
\[
\overline{V_{in}}^2 = \frac{\overline{V_{out}}^2}{A_v^2} = \left( 4kT \frac{g_{m1} + 1}{R_L} \right) \Delta f
\]
\[
\frac{\overline{I_{in}}^2}{\Delta f} = \left( \frac{g_{m1}^2 + g_{m2}^2}{g_{m2}} \right) 4kT \left( \frac{1}{g_{m2}} \right)^2
\]