4.1 a) $I_{C1} I_{C2} = I_{C3} I_{C4}$

$I_y I_x = I_2 I_o$ $\boxed{I_o = \dfrac{I_x I_y}{I_2}}$

b) $I_{C1} I_{C2} = I_{C3} I_{C4}$

$I_y I_x = I_o I_o$ $\boxed{I_o = \sqrt{I_x I_y}}$

c) $I_{C1} I_{C4} = I_{C3} I_{C2}$ $I_o = I_{C3} - I_{C4}$

$(1-x) I_x I_{C4} = I_{C3} (1+x) I_x$ $I_y = I_{C3} + I_{C4}$

$$I_{C4} = I_{C3} \frac{(1+x)\cancel{I_x}}{(1-x)\cancel{I_x}}$$

$$I_y = (I_o + I_{C4}) + I_{C4}$$

$$I_y = I_o + 2 I_{C4}$$

$$I_{C4} = (I_o + I_{C4}) \frac{(1+x)}{(1-x)}$$

$$I_y = I_o - \frac{(1+x)}{x} I_o$$

$$(1-x) I_{C4} - (1+x) I_{C4} = (1+x) I_o$$

$$I_y = \cancel{I_o} - \frac{1}{x} I_o \cancel{- I_o}$$

$$-2x I_{C4} = (1+x) I_o$$

$$2 I_{C4} = -\frac{(1+x)}{x} I_o$$

$$\boxed{I_o = -x I_y}$$

4.2 Gain = 10 dB

$HD_3 = -40\,dB$ @ $-15\,dBm$

a) $20\log(IM_3) = 20\log(3\,HD_3)$

$IM_3 = HD_3 + 9.5\,dB = \boxed{30.5\,dB}$

b) Input power measured in 50Ω

$P = \dfrac{V_{RMS}^2}{50}$ $\quad V_{RMS} = \sqrt{50P}$

$V_1 = \sqrt{2}\,V_{RMS} = \sqrt{100P} = 56.2\,mV$

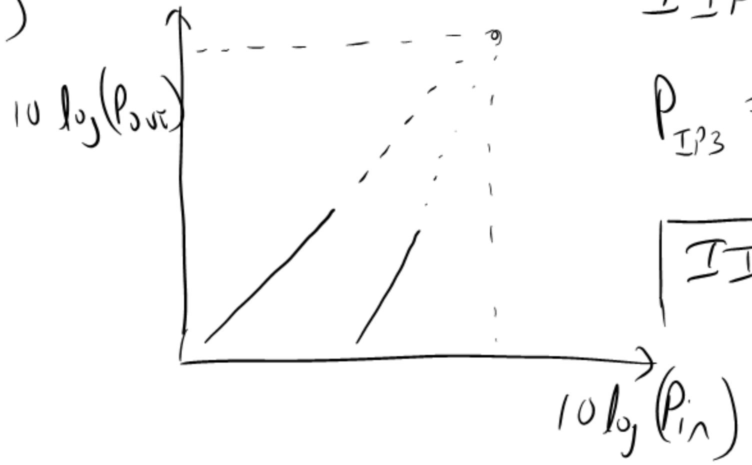
$20\log\alpha_1 = 10\,dB$ $\quad \boxed{\alpha_1 = 3.16}$

$HD_3 = \dfrac{1}{4}\dfrac{\alpha_3}{\alpha_1}V_1^2$ $\quad 20\log(HD_3) = -40\,dB$

$HD_3 = 0.01$

$\boxed{\alpha_3 = 40}$

c)



IIP3 $\quad V_{IIP} = \sqrt{\dfrac{4}{3}\left|\dfrac{\alpha_1}{\alpha_3}\right|}$

$P_{IP3} = \dfrac{1}{2}\dfrac{V_{IIP}^2}{50}$

$$\boxed{\begin{aligned} IIP3 &= 0.2\,dBm \\ OIP3 &= IIP3 + Gain \\ &= 10.2\,dBm \end{aligned}}$$

4.3 Gain $= 10dB$

$HD_2 = -20dB$ @ $-20dBm$, 50Ω

a) DC component from $\alpha_2 V_{in}^2$ term

DC value $= \dfrac{\alpha_2 V_1^2}{2}$

$$HD_2 = \frac{1}{2}\frac{\alpha_2}{\alpha_1}V_1 = \frac{1}{\alpha_1 V_1}\frac{\alpha_2 V_1^2}{2} = \frac{1}{V_{01}}\frac{\alpha_2 V_1^2}{2}$$

$$\frac{\alpha_2 V_1^2}{2} = HD_2 \cdot V_{01} \qquad V_{01} = \sqrt{100\, P_o}$$

$$P_o = -20dBm + \underbrace{10dB}_{\text{gain}} = -10dBm = -40dBW$$

$$\text{DC off.} = -20dB + (-40dBW) + \underbrace{10\log(100)}_{2}$$

$$\boxed{\text{DC off} = 10mV}$$

b) Reduce $\dfrac{\alpha_2 V_1^2}{2}$ by 10x (V_1^2 reduce by 10dB)

$$\boxed{P_{in} = -30dBm}$$

$$4.4 \quad THD = \frac{\text{Sum harmonic power}}{\text{power of fund}}$$

$$HD2 = \frac{\text{Amp. 2nd harm}}{\text{Amp. Fund}}$$

Ratio of powers $= (HD2)^2$

$(HD3)^2$

$(HD4)^2$

$\vdots$

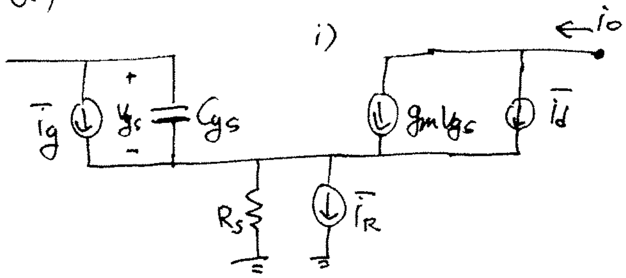
$$THD = \frac{P^{2nd} + P^{3rd} + P^{4th}}{P^{fund}}$$

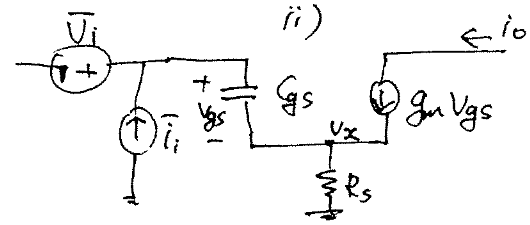
$$= \frac{P^{2nd}}{P^{fund}} + \frac{P^{3rd}}{P^{fund}} + \frac{P^{4th}}{P^{fund}}$$

$$THD = \underline{10^{-30/10} + 10^{-40/10} + 10^{-50/10}}$$

$$\boxed{THD = 0.11\ \%}$$

4.5. a)





1) short - input

i) $V_{gs} = (-\overline{i_g} - \overline{i_d} - g_m V_{gs} + \overline{i_s}) \cdot \left(R_s \, // \, \frac{1}{sC_{gs}}\right)$

$$\Rightarrow V_{gs} = \frac{R_s \, // \, \frac{1}{sC_{gs}}}{1 + g_m \left(R_s // \frac{1}{sC_{gs}}\right)} \cdot (-\overline{i_g} - \overline{i_d} + \overline{i_R}) = \frac{R_s}{1 + g_m R_s + sR_sC_{gs}} (-\overline{i_g} - \overline{i_d} + \overline{i_R})$$

$$i_o = g_m V_{gs} + i_d = \frac{g_m R_s}{1 + g_m R_s + sR_sC_{gs}} \, -\overline{i_g} + \frac{1 + sR_sC_{gs}}{1 + g_m R_s + sR_sC_{gs}} \overline{i_d} + \frac{g_m R_s}{1 + g_m R_s + sR_sC_{gs}} \, i$$

ii) KCL $\quad (V_i - V_x) \cdot sC_{gs} + g_m (V_i - V_x) = \frac{V_x}{R_s}$

$$V_x = \frac{g_m + sC_{gs}}{\frac{1}{R_s} + g_m + sC_{gs}} V_i$$

$$i_o = g_m (V_i - V_x) = g_m \cdot \frac{\frac{1}{R_s}}{\frac{1}{R_s} + g_m + sC_{gs}} \cdot V_i = \frac{g_m}{1 + g_m R_s + sR_sC_{gs}} V_i$$

by i), ii) $\quad V_i = -R_s \cdot \overline{i_g} + \frac{1 + sR_sC_{gs}}{g_m} \cdot \overline{i_d} + R_s \cdot \overline{i_R}$

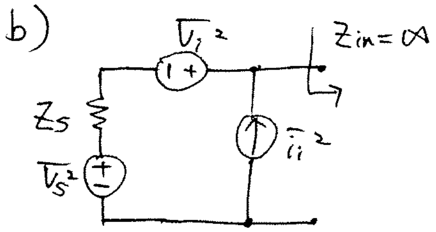
2) open - input

i) $i_o = g_m \cdot \left(-\overline{i_g} \cdot \frac{1}{sC_{gs}}\right) + \overline{i_d}$

ii) $i_o = g_m \cdot \overline{i_i} \cdot \frac{1}{sC_{gs}}$

$$\Rightarrow i_i = -\overline{i_g} + \frac{sC_{gs}}{g_m} \overline{i_d}$$

b)



If we use $Z_{in}=\infty$,

$$F = \frac{\overline{V_s^2} + \overline{|V_i + Z_S i_i|^2}}{\overline{V_s^2}}$$

($\because$ Noise Factor is independent of Z_{in})

- $V_i = -R_S\,\overline{i_g} + \dfrac{1+sR_SC_{gs}}{g_m}\,\overline{i_d} + R_S\cdot\overline{i_R}$
- $i_i = -\overline{i_g} + \dfrac{sC_{gs}}{g_m}\,\overline{i_d}$

if we assume $1 \ll sR_SC_{gs}$, $V_i \approx \underbrace{R_S\cdot i_i}_{V_c} + \underbrace{R_S\, i_R}_{V_u}$

V_c correlated with i_i $\quad$ V_u uncorrelated with i_i

$Z_c = R_S = \dfrac{V_c}{i_i}$

$$F = 1 + \frac{|Z_c+Z_S|^2\,\overline{i_i^2} + \overline{V_u^2}}{\overline{V_s^2}}$$

Let $G_n = \dfrac{\overline{i_i^2}}{4KT\Delta f}$, $R_u = \dfrac{\overline{V_u^2}}{4KT\Delta f}$, $R_{SS} = \dfrac{\overline{V_s^2}}{4KT\Delta f}$

Then, $F = 1 + \dfrac{R_u + |Z_c+Z_S|^2\cdot G_n}{R_{SS}}$

Similar with Lecture Note (Chap. 7. page 19),

Let $Z_c = R_c + jI_c$, $Z_S = R_{SS} + jI_S$

$$\begin{cases} I_{S,opt} = -I_c. \\ R_{SS,opt} = \sqrt{\dfrac{R_u}{G_n} + R_c^2} \end{cases}$$

- $R_u = R_S$
- $R_c = R_S$
- $G_n = \dfrac{\overline{\left|-i_g + \frac{sC_{gs}}{g_m}i_d\right|^2}}{4KT\Delta f} = \dfrac{\left|-c\cdot\frac{i_g}{i_d} + \frac{sC_{gs}}{g_m}\right|^2\cdot\overline{i_d^2} + (1-|c|^2)\cdot\overline{i_g^2}}{4KT\,\Delta f}$

$$= \left(|c|\cdot\frac{\omega C_{gs}}{g_{do}}\sqrt{\frac{\delta}{5\gamma}} + \frac{\omega C_{gs}}{g_m}\right)^2\cdot\gamma\cdot g_{do} + (1-|c|^2)\cdot\delta\cdot\frac{\omega^2 C_{gs}^2}{5g_{do}}$$

c) $Y_S = \dfrac{1}{50\,\Omega} \Rightarrow Z_S = R_{SS} = 50\,\Omega$.

$$F = 1 + \frac{R_S + |R_S+50|^2 G_n}{50}$$