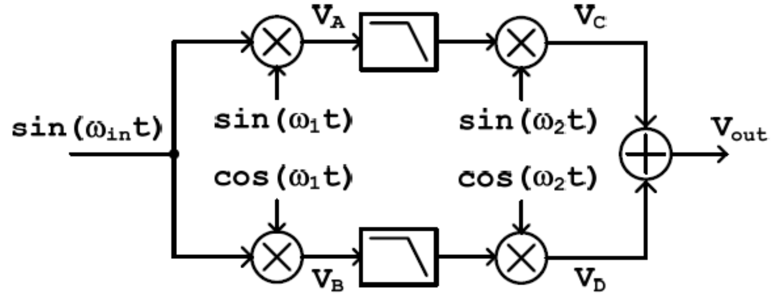
Problem 5.2: The block diagram below is for a single-sideband (SSB) AM modulator.



a) Assuming $\omega_2 \gg \omega_1 \gg \omega_{in}$, derive expressions for and sketch the spectrum of the signal at V_A, V_B, V_C, V_D, and V_{out}. The cutoff frequency of the low-pass filter is ω_1.

$$V_A = \sin(\omega_{in} t)\sin(\omega_1 t)$$

$$\boxed{V_A = \frac{1}{2}\left[\cos(\omega_{in} - \omega_1)t - \cos(\omega_{in} + \omega_1)t\right]}$$

$$V_B = \sin(\omega_{in} t)\cos(\omega_1 t)$$

$$\boxed{V_B = \frac{1}{2}\left[\sin(\omega_1 + \omega_{in})t + \sin(\omega_{in} - \omega_1)t\right]}$$

$$V_C = \left[\frac{1}{2}\cos(\omega_{in} - \omega_1)t\right]\left[\sin(\omega_2 t)\right]$$

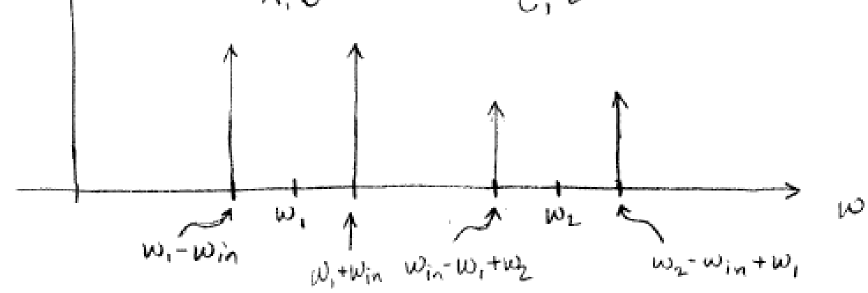
$$\boxed{V_C = \frac{1}{4}\left[\sin(\omega_{in} - \omega_1 + \omega_2)t + \sin(\omega_2 - \omega_{in} + \omega_1)t\right]}$$

$$V_D = \left[\frac{1}{2}\sin(\omega_{in} - \omega_1)t\right]\left[\cos(\omega_2 t)\right]$$

$$\boxed{V_D = \frac{1}{4}\left[\sin(\omega_{in} - \omega_1 + \omega_2)t + \sin(\omega_{in} - \omega_1 - \omega_2)t\right]}$$

$$V_{OUT} = V_C + V_D = \frac{1}{2}\sin(\omega_{in} - \omega_1 + \omega_2)t + 0$$

$$\boxed{V_{OUT} = \frac{1}{2}\sin(\omega_{in} - \omega_1 + \omega_2)t}$$



b) What is the frequency of the signal at the output?

$$\boxed{f_{out} = \frac{1}{2\pi}\left(\omega_{in} - \omega_1 + \omega_2\right)}$$

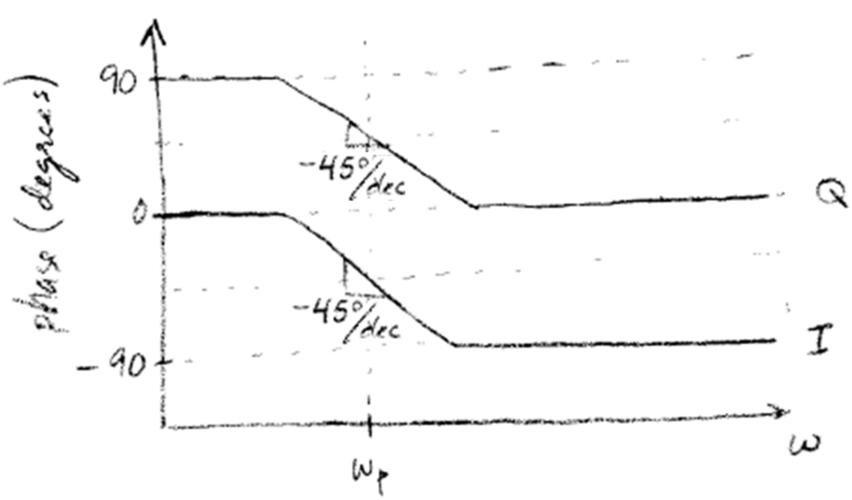
Problem 5.3: The RC-CR filter shown on the right can be used to generate quadrature (sin and cos) tones from a single oscillator.

a) Derive expressions for the phases of the two outputs V_I and V_Q. Sketch the two phases on a single graph.

$$V_I = V_{LO}\left(\frac{1}{1+sRC}\right) \Rightarrow \text{pole: } \omega_p = \frac{-1}{RC}$$

$$V_Q = V_{LO}\left(\frac{sRC}{1+sRC}\right) \Rightarrow \text{pole: } \omega_p = \frac{-1}{RC}$$

$$\text{zero: } \omega_z = 0$$



b) At what frequency are the amplitudes of V_I and V_Q equal?

$$|V_I| = V_{LO}\left|\frac{1}{1+j\omega RC}\right| = V_{LO}\left|\frac{1-j\omega RC}{1+\omega^2R^2C^2}\right|$$

$$|V_I| = V_{LO}\frac{1}{1+\omega^2R^2C^2}\left[\sqrt{1+\omega^2R^2C^2}\right]$$

$$|V_Q| = V_{LO}\left|\frac{j\omega RC}{1+j\omega RC}\right| = V_{LO}\left|\frac{\omega^2C^2R^2+j\omega RC}{1+\omega^2R^2C^2}\right|$$

$$|V_Q| = V_{LO}\frac{1}{1+\omega^2R^2C^2}\left[\sqrt{\omega^4C^4R^4+\omega^2R^2C^2}\right]$$

$$|V_Q| = V_{LO}\frac{\omega RC}{1+\omega^2R^2C^2}\left[\sqrt{\omega^2C^2R^2+1}\right]$$

$$|V_I| = |V_Q|$$

$$\sqrt{1+\omega^2R^2C^2} = \omega RC\sqrt{\omega^2C^2R^2+1}$$

$$\omega RC = 1$$

$$\omega = \frac{1}{RC}$$

$$\boxed{f = \frac{1}{2\pi RC}}$$

c) What is the amplitude of V_I and V_Q at the frequency found in part b) for an input $V_{LO} = V_A\cos(\omega_{LO}t)$?

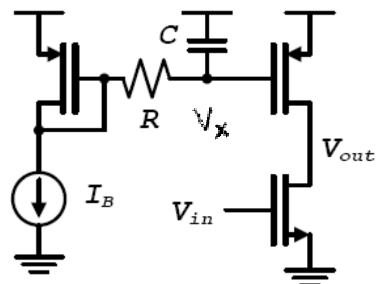
$$V_I = V_{LO}\left[\frac{1}{1+j\omega_{LO}RC}\right] \qquad V_Q = V_{LO}\left[\frac{j\omega_{LO}RC}{1+j\omega_{LO}RC}\right]$$

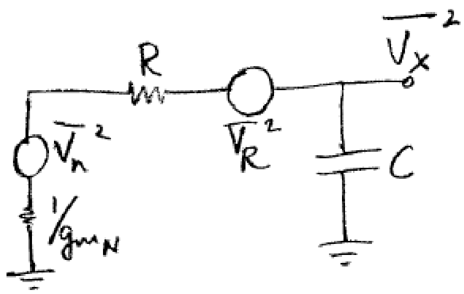
$$\omega_{LO} = \frac{1}{RC}$$

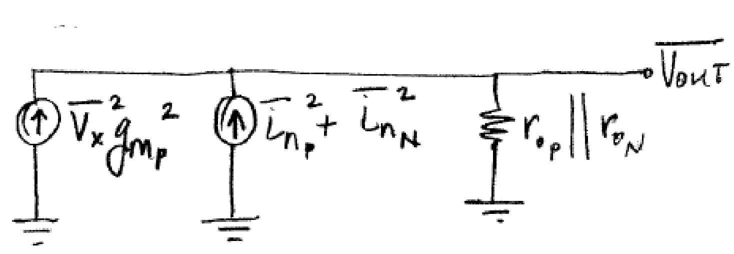
$$|V_I| = V_{LO}\left|\frac{1}{1+j}\right| \qquad |V_Q| = \left|\frac{j}{1+j}\right|$$

$$\boxed{|V_I| = \frac{\sqrt{2}}{2}V_{LO}} \qquad \boxed{|V_Q| = \frac{\sqrt{2}}{2}V_{LO}}$$

Problem 5.4: For this problem, use the circuit on the right. Assume that all transistors are matched, and that C is the only capacitance in the circuit. Consider only drain thermal noise in the FETs and thermal noise in the resistor. Find an expression for the frequency dependent input referred noise.







$$\overline{V_x^2} = \left(\overline{V_n^2} + \overline{V_R^2}\right)\left(\frac{1}{1+sC\left(R+\frac{1}{g_{mN}}\right)}\right)$$

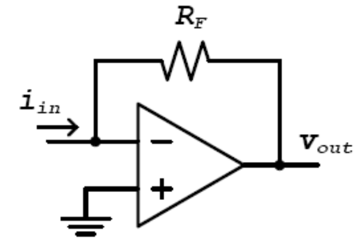
$$\overline{V_o^2} = \left[\overline{V_x^2}\, g_{mP}^2 + \overline{i_{nN}^2} + \overline{i_{nP}^2}\right]\left[r_{oN} \| r_{oP}\right]$$

$$\frac{\overline{V_o^2}}{\Delta f} = \left[4kT\left(\frac{\gamma}{g_{mN}} + R_F\right)\left(\frac{g_{mP}^2}{1+sC\left(R+\frac{1}{g_{mN}}\right)}\right) + 4kT\left[\left(\frac{\gamma}{g_m}\right)_P + \left(\frac{\gamma}{g_m}\right)_N\right]\right]\left[r_{oP} \| r_{oN}\right]^2$$

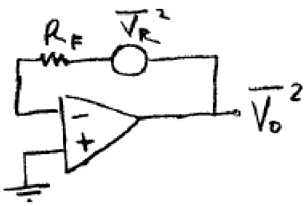
$$\overline{V_i^2} = \frac{\overline{V_o^2}}{A_v^2} = \frac{\overline{V_o^2}}{g_{mN}^2\left(r_{oP} \| r_{oN}\right)^2}$$

$$\boxed{\frac{\overline{V_i^2}}{\Delta f} = 4kT\frac{1}{g_{mN}^2}\left(\frac{\gamma}{g_{mN}} + R_F\right)\left[\frac{g_{mP}^2}{1+sC\left(R+\frac{1}{g_{mN}}\right)}\right] + 4kT\left[\left(\frac{\gamma}{g_m}\right)_P + \left(\frac{\gamma}{g_m}\right)_N\right]}$$

Problem 5.5: The problem looks at the noise performance of a transimpedance amplifier, commonly used when interfacing to photo diodes.



a) For this part, use the transimpedance amplifier shown on the right. Assume the amplifier is noiseless. Find an expression for the input referred noise current and output referred noise voltage considering only thermal noise in the resistor.



$$A_{I-V} = -R_F$$

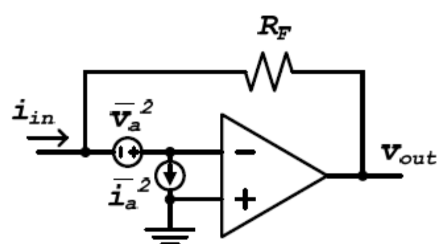
$$\overline{V_o^2} = \overline{V_R^2}$$

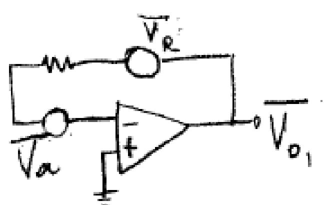
$$\boxed{\overline{V_o^2} = 4kTR_F\Delta f}$$

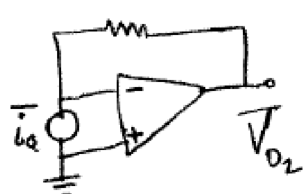
$$\overline{i_i^2} = \frac{\overline{V_o^2}}{A_{I-V}^2}$$

$$\boxed{\overline{i_i^2} = 4kT\frac{1}{R_F}\Delta f}$$

b) Now consider an amplifier with equivalent input noise generators v_a^2 and i_a^2. Find a new expression for the input referred noise current also considering thermal noise in the resistor.
Hint: remember that v_a and i_a may be correlated.







$$\overline{V_{o_1}} = \overline{V_a} + \overline{V_R} \qquad \overline{V_{o_2}} = \overline{i_a} R_F$$

$$\overline{V_o^2} = \left(\overline{V_{o_1}} + \overline{V_{o_2}}\right)^2$$

$$\overline{i_i^2} = \frac{\overline{V_o^2}}{R_F^2} = \overline{i_R^2} + \left(\overline{i_a} + \frac{\overline{V_a}}{R_F}\right)^2$$

$$\boxed{\overline{i_i^2} = 4kT\frac{1}{R_F}\Delta f + \left(\overline{i_a} + \frac{\overline{V_a}}{R_F}\right)^2}$$