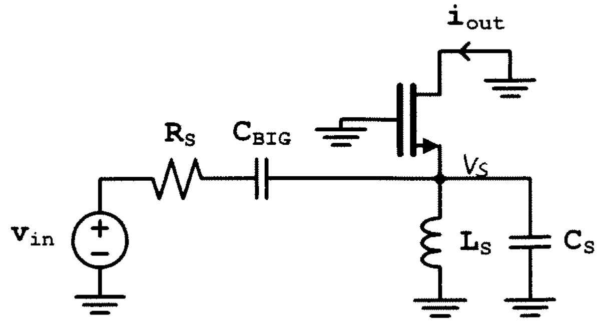
Problem 6.1: Use the common-gate amplifier below for this problem. Use the combined model for drain and gate noise, $\overline{i_{ndg}^2}$, derived in lecture, making the same assumptions on the small-signal model (no r_o, include C_{gs}, etc.). Assume C_{BIG} is a short at AC, and the circuit is operated at the self-resonant frequency $\omega_0 = 1/\sqrt{L_S(C_S + C_{gs})}$.



a) What is the small-signal transconductance $G_m = i_{out}/v_{in}$ of the circuit at $\omega = \omega_0$ assuming the transistor is biased so that $1/g_m = R_S$.

$$V_S = V_{in} \cdot \frac{\frac{1}{g_m} // sL_s // \frac{1}{s(C_{gs}+C_s)}}{R_s + \frac{1}{g_m} // sL_s // \frac{1}{s(C_{gs}+C_s)}}$$

$$= V_{in} \cdot \frac{\frac{1}{g_m}}{R_s + \frac{1}{g_m}} = \frac{1}{2} V_{in}$$

$$i_{out} = g_m \cdot (0 - V_s) = g_m \cdot -\frac{1}{2} V_{in} \qquad \therefore G_m = \frac{i_{out}}{V_{in}} = -\frac{1}{2} g_m$$

b) Derive expressions for η and Z_{gsw} for the combined drain and gate noise, $\overline{i_{ndg}^2}$.

$$Z_{gs} = \frac{1}{sC_{gs}} // \frac{Z_{deg} + Z_g}{1 + g_m Z_{deg}}$$

$$\cdot Z_g = 0 \qquad \cdot Z_{deg} = R_s // sL_s // \frac{1}{sC_s} = \frac{1}{\frac{1}{R_s} + \frac{1}{j\omega L_s} + j\omega C_s} = \frac{j\omega L_s R_s}{R_s - \omega^2 C_s L_s R_s + j\omega L_s}$$

$$\therefore Z_{gs} = \frac{1}{j\omega C_{gs} + \frac{R_s - \omega^2 C_s L_s R_s + j\omega L_s + g_m j\omega L_s R_s}{j\omega L_s R_s}} = \frac{j\omega L_s R_s}{\cancel{-\omega^2 C_{gs} L_s R_s} \cancel{+ R_s} \cancel{- \omega^2 C_s L_s R_s} + j\omega L_s + j\omega L_s \cdot R_s \cdot g_m} = \frac{j\omega L_s R_s}{2j\omega L_s} = \frac{R}{2}$$

$$\eta = 1 - \frac{g_m Z_{deg}}{Z_{deg} + Z_g} \cdot Z_{gs} = 1 - g_m \frac{R_s}{2} = \frac{1}{2} \quad (\because 1/g_m = R_s)$$

$$Z_{gsw} = \omega C_{gs} Z_{gs} = \frac{\omega C_{gs} R_s}{2}$$

$$\overline{i_{ndg}^2} = \overline{i_{nd}^2}\left(|\eta|^2 + 2\,\mathrm{Re}\{c\,\chi_d \eta^* Z_{gsw}\} + \chi_d^2 |Z_{gsw}|^2\right) = \overline{i_{nd}^2}\left(\frac{1}{4} + \chi_d^2 \cdot \frac{R_s^2}{4} \cdot \omega^2 C_{gs}^2\right)$$

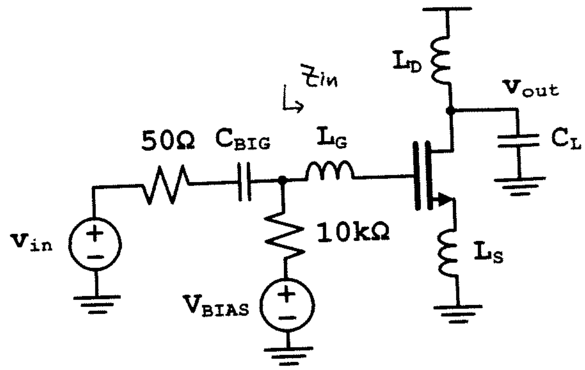
c) Derive an expression for the noise factor of the amplifier using your results from b).

$$F = \frac{\overline{e_n^2} \cdot (-\frac{1}{2} g_m)^2 + \overline{i_{nd}^2}(1/4 + \chi_d^2 R_s^2 \omega^2 C_{gs}^2/4)}{\overline{e_n^2} \cdot (-\frac{1}{2} g_m)^2} = 1 + \frac{4kT\gamma g_{d0}(\frac{1}{4} + \chi_d^2 R_s^2 \omega^2 C_{gs}^2/4)}{4kT\Delta f R_s \cdot \frac{1}{4} g_m^2} = 1 + \frac{\gamma}{\alpha} + \frac{\alpha\delta\omega^2 C_{gs}^2}{5 g_m^2}$$

d) Assuming c is negative and imaginary, $\omega = \omega_0 = 1/\sqrt{L_S(C_S + C_{gs})}$, $Q_{in} = \frac{1}{2} R_S (C_S + C_{gs})\omega_0$, and $\omega_t = g_m/C_{gs}$, simplify your expression from c) into a form containing these quantities and device constants.

$$F = 1 + \frac{\gamma}{\alpha} + \frac{\alpha\delta}{5} \frac{\omega_0^2}{\omega_t^2}$$

Problem 6.2: Use the amplifier below for this problem. Consider only C_{gs} and g_m for the FET, neglect all other small signal parameters. Assume the input and output resonances are tuned to 1GHz, and that C_{BIG} is a short circuit at 10GHz.



a) Assume that Q=10 for all inductors at 1GHz, and the inductors are modeled as a series R and L. Given that $\omega_t = 2\pi 3.2 GHz$ and $g_m = 10 mMho$, calculate the values of L_G and L_s to provide a 50Ω match to the source.

$$W_t = 2\pi \cdot 3.2 \cdot 10^9 = \frac{g_m}{C_{gs}}$$

$$C_{gs} = \frac{10 \times 10^{-3}}{2\pi \cdot 3.2 \times 10^9}$$

$$Z_{in} = jwL_G + \frac{wL_G}{Q} + \frac{1}{jwC_{gs}} + \frac{jwC_{gs} + g_m}{jwC_{gs}} \cdot \left(jwL_s + \frac{wL_s}{Q}\right)$$

$$= \underbrace{\left(\frac{wL_G}{Q} + \frac{wL_s}{Q} + \frac{g_m}{C_{gs}} L_s\right)}_{=50} + j\underbrace{\left(wL_G + wL_s - \frac{1}{wC_{gs}} - \frac{g_m L_s}{C_{gs} Q}\right)}_{=0}$$

$$\begin{cases} \frac{w}{Q}(L_G + L_s) + w_t L_s = 50 & -(1) \\ w(L_G + L_s) - \frac{1}{wC_{gs}} - \frac{w_t}{Q} L_s = 0 & -(2) \end{cases}$$

by (1), (2) $\quad L_G = 50.34\,nH, \; L_s = 0.886\,nH$

b) Assuming $V_{GS} - V_{th} = 150mV$, calculate the bias current in the FET and the voltage drop across L_S.

$$I_D = \frac{1}{2} g_m \cdot (V_{GS} - V_{th}) = \frac{1}{2} \cdot 10m \cdot 150m = 0.75\,mA$$

$$R_{Ls} = \frac{w_o L_s}{Q} = \frac{2\pi \cdot 10^9 \cdot 0.886nH}{10} = 0.5567\,\Omega$$

$$V_{Ls} = I_D \cdot R_{Ls} = 0.4175\,mV$$

c) Assume this amplifier is loaded by an identical FET, therefore $C_L = C_{gs}$. Calculate the value of L_D required to resonate out this capacitance at 1GHz. Is this a reasonable value?

$$w_o = \frac{1}{\sqrt{L_D C_{gs}}} \Rightarrow L_D = \frac{1}{w_o^2 C_{gs}} = \frac{1}{w_o^2 \cdot \frac{g_m}{w_t}} = 50.93\,nH$$

d) What is the overall voltage gain of your amplifier at 1GHz? Provide a numerical value (still neglecting r_o of the FET) with Q=10 for all inductors at 1GHz.

$$A_v(\omega_o) = -g_m \cdot Z_{out}(\omega_o)$$

$$= -g_m \, (1+Q^2) R_{Lo} \cdot Q_{in}$$

$$= -g_m \, (1+Q^2) \frac{\omega_o L_o}{Q} \cdot Q_{in}$$

$$\approx -103.42 \text{ V/V} \qquad \left(Q_{in} = \frac{1}{2 \omega_o C_{gs} R_s} = 3.2 \right)$$

e) What is the noise factor of your amplifier. Assume $\gamma = 3$, $\delta = 6$, $\alpha = 0.75$, and $c = -0.55j$.

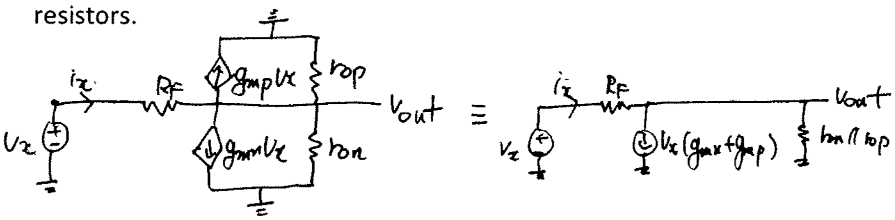
$$\chi_d = \alpha \sqrt{\frac{\delta}{5\gamma}} \approx 0.474, \qquad Q_{in} = \frac{1}{2 \omega_o C_{gs} R_s} \approx 3.2$$

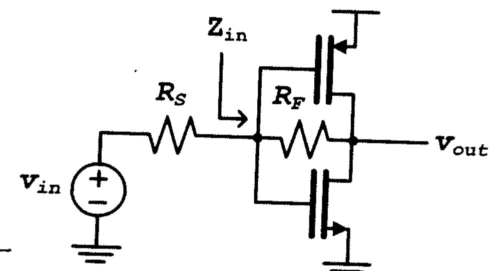
$$F = 1 + \frac{\omega_o}{\omega_t} \cdot \frac{\gamma}{\alpha} \cdot \frac{1}{2Q_{in}} \left(1 - 2|c|\chi_d + (4Q_{in}^2 + 1)\chi_d^2 \right)$$

$$= 2.936$$

$$NF = 4.68 \text{ dB}$$

Problem 6.3: Derive expressions for the input impedance Z_{in}, voltage gain v_{out}/v_{in}, and noise factor of the following circuit seen in lecture. Consider only drain thermal noise and thermal noise in the resistors.





$$V_{out} = -\frac{R_F}{R_F + r_{on}//r_{op}} \cdot (g_{mn}+g_{mp})V_x \cdot r_{op}//r_{on} + V_x \cdot \frac{r_{on}//r_{op}}{R_F + r_{on}//r_{op}}$$

$$I_x = V_x \cdot (g_{mn}+g_{mp}) + \frac{V_{out}}{r_{on}//r_{op}} = V_x (g_{mn}+g_{mp}) - \frac{R_F}{R_F + r_{on}//r_{op}} V_x (g_{mn}+g_{mp}) + \frac{V_x}{R_F + (r_{on}//r_{op})}$$

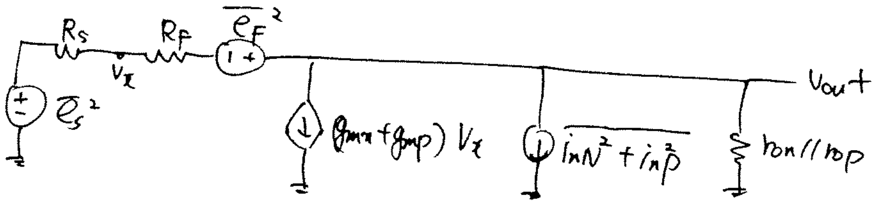
- $$Z_{in} = \frac{V_x}{I_x} = \frac{1}{g_{mn}+g_{mp} - \frac{R_F (g_{mn}+g_{mp})}{R_F + r_{on}//r_{op}} + \frac{1}{R_F + r_{on}//r_{op}}}$$

$$\approx \frac{1}{g_{mn}+g_{mp}} \quad \text{for } r_{on}//r_{op} \gg R_F$$

- $$A_v = \frac{V_{out}}{V_{in}} = \frac{V_x}{V_{in}} \cdot \frac{V_{out}}{V_x} = \frac{Z_{in}}{R_s + Z_{in}} \left(-R_F(g_{mn}+g_{mp}) + 1 \right) \cdot \frac{r_{on}//r_{op}}{R_F + (r_{on}//r_{op})}$$

$$\approx \frac{\frac{1}{g_{mn}+g_{mp}}}{R_s + \frac{1}{g_{mn}+g_{mp}}} \cdot -R_F(g_{mn}+g_{mp}) = -\frac{(g_{mn}+g_{mp}) \cdot R_F}{1 + (g_{mn}+g_{mp})R_s}$$

: Noise Factor



· Find $\dfrac{V_{out}}{e_F}$: ignore other source.

KCL at V_{out}:

$$\frac{V_{out}-e_F}{R_s+R_F} + (g_{mn}+g_{mp})\cdot(V_{out}-e_F)\cdot\frac{R_s}{R_s+R_F} + \frac{V_{out}}{r_{on}//r_{op}} = 0$$

$$\Rightarrow \frac{V_{out}}{e_F} = \left(\frac{1+(g_{mn}+g_{mp})R_s}{R_s+R_F} + \frac{1}{r_{on}//r_{op}}\right) \Big/ \frac{1+(g_{mn}+g_{mp})R_s}{R_s+R_F}$$

$$= 1 + \frac{R_s+R_F}{(1+(g_{mn}+g_{mp})R_s)\cdot r_{on}//r_{op}} \approx 1$$

· Total Noise, $\overline{e_{total}}^2 = Av^2\cdot\overline{e_s}^2 + \overline{e_F}^2 + (\overline{i_{nN}^2}+\overline{i_{nP}^2})\cdot\left((R_s+R_F)//r_{on}//r_{op}//\frac{R_s+R_F}{R_s}\cdot\frac{1}{g_{mn}+g_{mp}}\right)$

$$\therefore F = \frac{Av^2\cdot\overline{e_s}^2 + \overline{e_F}^2 + (\overline{i_{nN}^2}+\overline{i_{nP}^2})\cdot R_F^2/(1+(g_{mn}+g_{mp})R_s)^2}{Av^2\cdot\overline{e_s}^2}$$

$$\left(\because (R_s+R_F)//r_{on}//r_{op}//\frac{R_s+R_F}{(g_{mn}+g_{mp})R_s} \approx \frac{R_F}{(1+(g_{mn}+g_{mp})R_s)}\right)$$

$$= 1 + \frac{4kT\Delta f R_F + 4kT\Delta f(\gamma_n g_{don}+\gamma_p g_{dop})\cdot R_F^2/(1+(g_{mn}+g_{mp})R_s)^2}{\frac{(g_{mn}+g_{mp})^2\cdot R_F^2}{(1+(g_{mn}+g_{mp})R_s)^2}\cdot 4kT\Delta f R_s}$$

$$= 1 + \frac{(1+(g_{mn}+g_{mp})R_s)^2}{(g_{mn}+g_{mp})^2\cdot R_s R_F} + \frac{\gamma_n g_{don}+\gamma_p g_{dop}}{(g_{mn}+g_{mp})^2\, R_s}$$

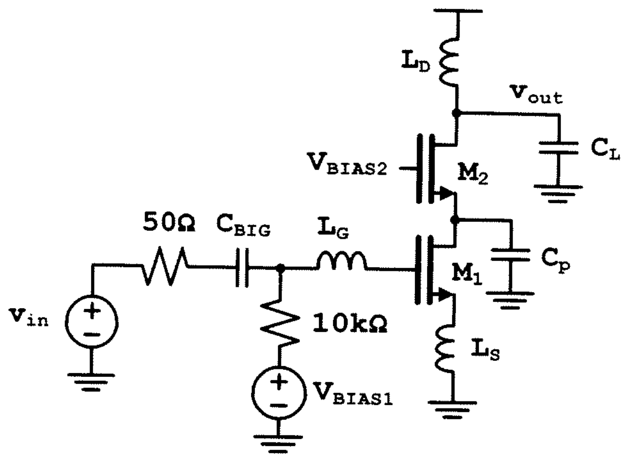
if input is matched, i.e $\dfrac{1}{g_{mn}+g_{mp}} = R_s$

$$F = 1 + \frac{4}{(g_{mn}+g_{mp})^2\cdot R_s R_F} + \frac{1}{(g_{mn}+g_{mp})R_s}\cdot\frac{\gamma_n g_{don}+\gamma_p g_{dop}}{g_{mn}+g_{mp}}$$

$$= 1 + \frac{4}{(g_{mn}+g_{mp})R_F} + \frac{\gamma_n g_{don}+\gamma_p g_{dop}}{g_{mn}+g_{mp}}$$

Problem 6.4: Use the cascode LNA shown below.

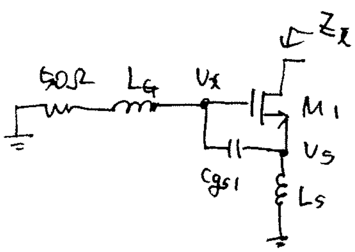
a) Derive an expression for the output-referred short-circuit noise current contributed from the drain thermal noise of transistor M2 neglecting r_o for both devices.



$$\overline{i_o^2} = \overline{i_{n2}^2} \cdot \left| \frac{\frac{1}{g_{m2}}}{\frac{1}{s(C_p + C_{gs2})} + \frac{1}{g_{m2}}} \right|^2$$

$$\overline{i_o^2} = 4kT(\gamma_2 g_{d0_2}) \cdot \left| \frac{s(C_p + C_{gs2})}{s(C_p + C_{gs2}) + g_{m2}} \right|^2 \Delta f$$

b) Now including r_{o1}, what is the impedance seen looking into the drain of M1 at the resonant frequency of the input matching circuit? Assume infinite Q in the inductors, and C_{BIG} is a short.



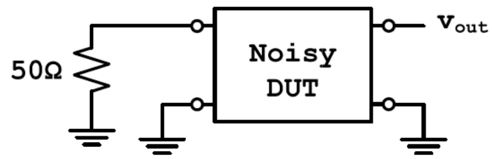
$$Z_x = \left[1 + \left(1 - \frac{V_x}{V_s}\right) \cdot g_m Z_s\right] r_{o1}$$

where $Z_s = \left(50 + sL_G + \frac{1}{sC_{gs1}}\right) // sL_s$

$$\frac{V_x}{V_s} = \frac{50 + sL_G}{50 + sL_G + \frac{1}{sC_{gs}}}$$

$$\therefore Z_x = \left[1 + \frac{\frac{1}{sC_{gs}}}{50 + sL_G + \frac{1}{sC_{gs}}} \cdot g_m \cdot \left(50 + sL_G + \frac{1}{sC_{gs1}}\right) // sL_s\right] \cdot r_{o1}$$

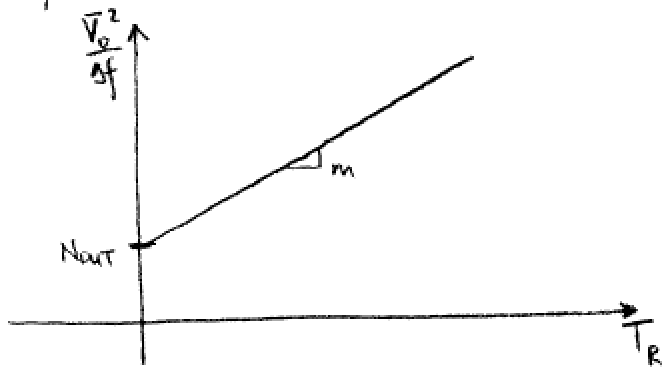
Problem 6.4: This question concerns the measurement of noise figure of an arbitrary device under test (DUT).



a) Assume that the temperature of the 50Ω source resistor can be precisely controlled. The noisy DUT has voltage gain A(s) and contributes a mean-square output referred noise voltage N_{DUT} in a given bandwidth Δf. Sketch a graph of the total mean square noise voltage measured at the output as a function of source resistor temperature. What is the slope of the line in V^2/T?

$$\frac{\overline{V_o^2}}{\Delta f} = 4kT_R R_s A^2 + N_{DUT}$$

$$\boxed{m = 4kTA^2}$$



b) What is the significance of the intercept of this graph at T=0 K?

At T = 0K, the output mean square noise voltage is equal to N_{DUT}.

c) Propose a technique to measure the noise figure (factor) of the DUT by only varying the temperature of the source. How many temperatures do you need? What if temperature control of R_S isn't available, but it's temperature is known. How could you measure noise figure in this case?

(a) 2 measures : $(T_{R_1}, \overline{V_{o_1}^2})$ and $(T_{R_2}, \overline{V_{o_2}^2})$

$$m = \frac{\overline{V_{o_2}^2} - \overline{V_{o_1}^2}}{T_{R_2} - T_{R_1}} \qquad \overline{V_{DUT}^2} = \overline{V_{o_1}^2} - mT_{R_1} = \overline{V_{o_2}^2} - mT_{R_2}$$

$$\boxed{F = 1 + \frac{\overline{V_{DUT}^2}}{mR_s} = 1 + \frac{\overline{V_{o_1}^2}}{mR_s} - \frac{T_{R_1}}{R_s} = 1 + \frac{\overline{V_{o_2}^2}}{mR_s} - \frac{T_{R_2}}{R_s}}$$

(b) 1 measure : $(T_{R_1}, \overline{V_{o_1}^2})$

$$N_{OUT} = N_{R_s} G_p + N_{DUT} \Longrightarrow N_{DUT} = \underset{\text{measured}}{N_{OUT}} - \underset{\text{calculated}}{N_{R_s}} \underset{\text{measured}}{G_p}$$

N_{OUT} = output noise power
N_{R_s} = R_s noise power
G_p = power gain

$$\boxed{F = 1 + \frac{N_{DUT}}{N_{R_s} G_p} = \frac{N_{OUT}}{N_{R_s} G_p}}$$