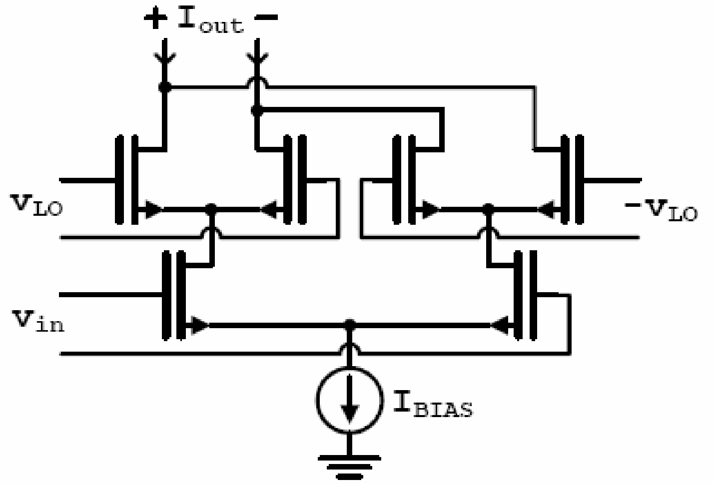
Problem 7.1: For this problem, assume the LO is a square wave at 900MHz, and the RF signal is a tone at 1GHz.



a) Considering only the first harmonic component of the LO signal, what frequencies appear at the output of the mixer? Assume the mixer is perfectly balanced.

$(900 + 1000)\,MHz = \boxed{1900\,MHz}$

$(1000 - 900)\,MHz = \boxed{100\,MHz}$

b) Derive an expression for the conversion gain of the mixer from the RF signal to the desired component at IF.

$$G_c = \frac{\text{IF amplitude}}{\text{RF amplitude}} = \boxed{\frac{2}{\pi} g_m}$$

c) Now introduce a phase offset between the LO applied to the left and right mixers of $\Delta\theta$. Assume $\Delta\theta$ is small. Derive an expression for the output amplitude of the LO leakage at 900MHz as a result of the offset.

Hint: $cos(A + \Delta) \approx cos(A) - \Delta sin(A)$ for small Δ.

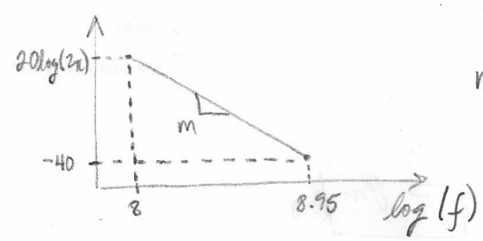
Assume $V_{in} = \frac{i_{RF}}{g_m} = 0$ and consider only 1st harmonic

$i_{IF}^+ = I_{Bias}\left(\frac{4}{\pi}\cos \omega_{LO} t\right) \quad i_{IF}^- = I_{Bias}\left(\frac{4}{\pi}\cos(\omega_{LO} + \Delta)t\right) \approx \frac{4 I_{Bias}}{\pi}\left[\cos(\omega_{LO}t) - \Delta \sin(\omega_{LO}t)\right]$

$$\boxed{I_{out} = i_{IF}^+ - i_{IF}^- = \frac{4}{\pi} I_{Bias}\, \Delta \sin(\omega_{LO} t)}$$

d) Using your expression for conversion gain from part b), find an expression for the ratio of LO leakage amplitude to IF signal amplitude in dBc. Assuming $G_m v_{in} = 1\%$ of I_{BIAS} for good linearity and $\Delta\theta = 2\pi/100$ (1%), what is the required slope of the IF filter in dB/decade to attenuate the LO signal to -40dBc below the IF signal level?

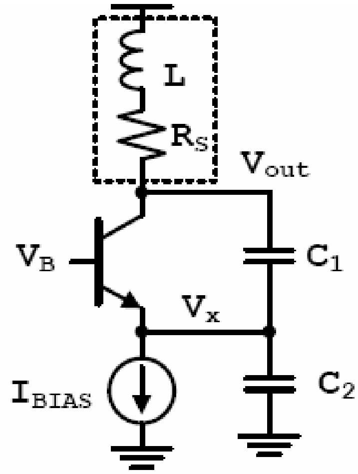
$$\frac{\frac{4}{\pi} I_{Bias}\,\Delta}{\frac{4}{\pi} g_m V_{in}} = \frac{I_{Bias}\,\frac{2\pi}{100}}{(.01)(I_{Bias})} = 2\pi$$



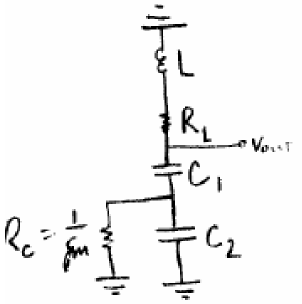
$$m = \frac{20\log(2\pi) - (-40)}{8 - 8.95}$$

$$\boxed{m = -58.65\ dB/dec}$$

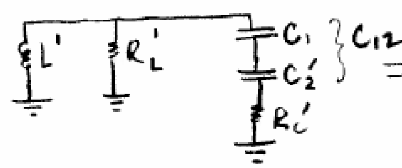
Problem 7.2: You may neglect r_o and base current ($\beta = \infty$) for this problem.



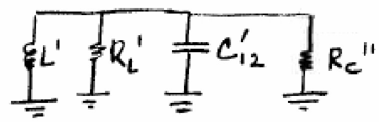
a) Simplify L, C_1, and C_2 into an equivalent parallel RLC tank. Use the series-to-parallel resistance transformations discussed in lecture to find an expression for the equivalent loss R_{eq} of the parallel tank. Assume a series loss in the inductor R_S, and an inductor $Q_L = 10$. Do not neglect the impedance on node V_x in your expression for R_{eq}.



$\Longrightarrow$



$\Longrightarrow$



$$L' = L\left(\frac{Q_L^2+1}{Q_L^2}\right)$$

$$R_L' = R_L(Q^2+1)$$

$$Q_2 = \omega R_c C_2 = \frac{\omega C_2}{g_m}$$

$$C_2' = C_2\left(\frac{Q_2^2}{Q_2^2+1}\right)$$

$$R_c' = R_c\left(\frac{1}{Q_2^2+1}\right)$$

$$Q_{12} = \frac{1}{\omega R_c' C_{12}}, \quad C_{12} = \frac{C_1 C_2'}{C_1+C_2}$$

$$C_{12}' = C_{12}\left(\frac{Q_{12}^2+1}{Q_{12}^2}\right)$$

$$R_c'' = R_c'(Q_{12}^2+1)$$

$\rightarrow$ Assuming $Q_2^2 \gg 1$ we have

$$Q_{12} \approx \frac{\omega C_2}{g_m}\left(\frac{C_1+C_2}{C_1}\right) \qquad R_c' \approx \frac{g_m}{\omega^2 C_2^2}$$

therefore,

$$C_{12}' = \frac{C_1 C_2}{C_1+C_2}\left(1 + \frac{g_m C_1}{\omega C_2 (C_1+C_2)}\right)$$

and

$$R_c'' = \frac{1}{g_m}\left(\frac{C_1+C_2}{C_1}\right)^2$$

$$Y_{eq} = \frac{-j}{\omega L'} + \frac{1}{R_L'} + j\omega C_{12}' + \frac{1}{R_c''} = \left(\frac{1}{R_L'} + \frac{1}{R_c''}\right) + j\left(\omega C_{12}' - \frac{1}{\omega L'}\right)$$

$$\boxed{R_{eq} = \mathrm{Re}\left(Y_{eq}^{-1}\right) \approx \left(\frac{1}{R_L'} + \frac{1}{R_c''}\right)^{-1}} \quad \text{for } \omega = \omega_{osc}$$

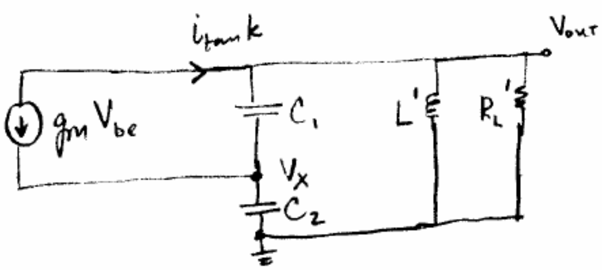
where $L' = \frac{101}{100} L \approx L \qquad R_L' = 101 R_L \qquad R_c'' = \frac{1}{g_m}\left(\frac{C_1+C_2}{C_1}\right)^2$

b) Derive an expression for the oscillation frequency.

$$f_{osc} = \frac{1}{2\pi}\frac{1}{\sqrt{L'C_{12}'}} \approx \frac{1}{2\pi}\frac{1}{\sqrt{L\,C_{12}}} \qquad \text{for} \quad Q_{12}^2 \gg 1 \ \text{and} \ Q_L^2 \gg 1$$

$$\boxed{f_{osc} \approx \frac{1}{2\pi}\sqrt{\frac{C_1+C_2}{L\,C_1\,C_2}}}$$

c) Now incorporating the small-signal model of the BJT, derive an expression for I_{BIAS} that biases the oscillator just at the edge of oscillation. To simplify your expression, use the definition $n = C_1/(C_1+C_2)$.



$$V_{be} = -V_x = -V_{out}\left(\frac{C_1}{C_1+C_2}\right)$$

$$i_{tank} = g_m V_x = g_m V_{out}\left(\frac{C_1}{C_1+C_2}\right)$$

$$\frac{V_{out}}{i_{tank}} = \frac{(C_1+C_2)}{g_m C_1} = \frac{1}{g_m n}$$

For oscillation

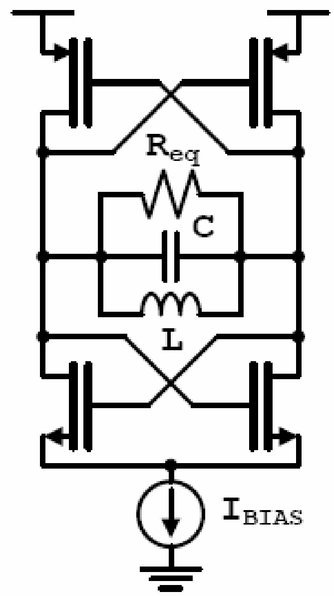
$$\frac{1}{n g_m} = R_{eq} = \frac{\left(\cancel{\frac{1}{R_L'}} + n^2 g_m\right)}{\left(\frac{1}{R_L'} + n^2 g_m\right)^2} \qquad \text{for} \quad \omega = \omega_{osc}$$

$$g_m = \frac{1}{(n-n^2)R_L'} = \frac{1}{(n-n^2)(Q_L^2+1)R_L}$$

$$g_m = \frac{I_{Bias}}{V_T} = \frac{1}{n(1-n)(Q_L^2+1)R_L}$$

$$\boxed{I_{Bias} = \frac{V_T}{n(1-n)(Q_L^2+1)R_L}} \qquad \text{where} \quad n = \frac{C_1}{C_1+C_2}, \quad Q_L = 10, \quad V_T = \frac{kT}{q}$$

Problem 7.3: For this problem, assume L and C are lossless, and all losses are modeled by R_{eq}.

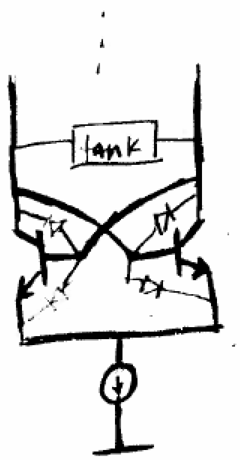


a) Derive and expression for the differential amplitude of the fundamental oscillation across the tank. Assume all higher-order harmonics are perfectly filtered by the tank.

$$V_{osc} \approx I_{Bias} (R_{eq}) \left(\frac{4}{\pi} \right) \cos(\omega_o t)$$

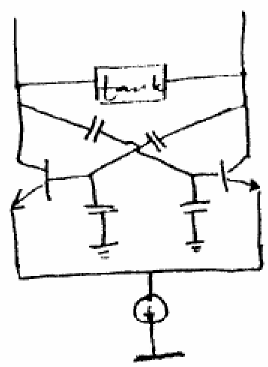
$$\boxed{|V_{osc}| \approx \left(\frac{4}{\pi} \right) \left(R_{eq} \cdot I_{Bias} \right)}$$

b) Replace the NMOS devices with NPNs. Explain how the amplitude is limited by the parasitic diodes in the BJTs. How would you modify the circuit to eliminate this limitation?



→ Amplitude is limited due to current path through parasitic diodes.

→ To eliminate this effect you can add voltage dividers using capacitors

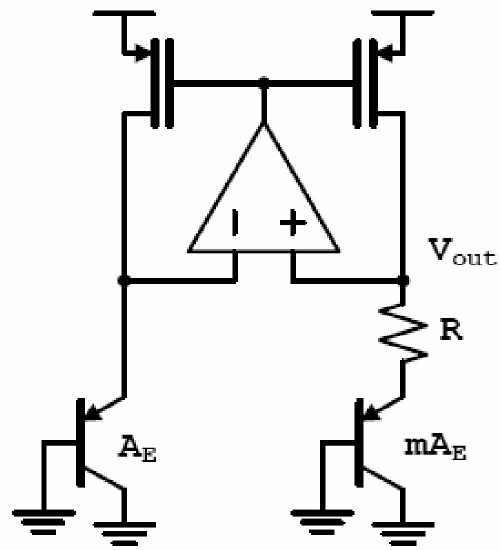


→ The problem with this is that you will now need a biasing circuit for the BJT's

→ You may also use buffers to reduce the limitation due to parasitic diodes.

Problem 7.4: This problem refers to the bandgap reference and corresponding expression for $\partial V_{BE}/\partial T$ shown in Lecture 22.

a) Assuming V_{G0} = 1.2V, V_{BE} = 750mV at 300K, and from simulation, the exponent of I_S temperature dependency r = 2. What is the value of n, the exponent of I_C temperature dependency, determined by the circuit on the right?



$$V_{BE1} = V_{BE2} + I_c R$$

$$I_c R = \Delta V_{BE} = V_T \ln m = \frac{kT}{q} \ln m$$

$$I_c = \frac{kT}{qR} \ln m \Rightarrow I_c \propto T^1$$

Therefore, $\boxed{n = 1}$

b) Using your answer from part, calculate $\partial V_{BE}/\partial T$ at 300K and determine the value of m (BJT area scaling ratio) to cancel this slope.

$$\frac{dV_{BE}}{dT} = -\frac{V_{G0} - V_{BE} + (r-n)V_T}{T} = -\frac{1.2 - .75 + (2-1)(.026)}{300} = \boxed{-1.587\ mV/k}$$

$$\frac{d\Delta V_{BE}}{dT} = \frac{k}{q} \ln m = 1.587m \Rightarrow \underline{\ln m = 18.4}$$

$$\boxed{m \approx 9.76 \times 10^7}$$

c) Now assume the resistor has a temperature dependency, typically proportional to $\sqrt{T}$. What is the new value of n, the exponent of I_C temperature dependency, and new value of $\partial V_{BE}/\partial T$?

From part a) $I_c = \frac{kT}{qR} \ln m \Rightarrow I_c \propto \frac{T^1}{T^{1/2}}$

$$\underline{I_c \propto T^{1/2}} \Rightarrow \boxed{n = 1/2}$$

$$\frac{dV_{BE}}{dT} = -\frac{V_{G0} - V_{BE} + (r-n)V_T}{T} = \boxed{-1.63\ mV/K}$$

d) Using your value of m from part b), and the value of $\partial V_{BE}/\partial T$ from part c) calculated at 300K, what is the error voltage at V_{out} at 0°C and 85°C, the extremes of the commercial temperature range?

$$V_{out_{error}} = V_{be} + \Delta V_{be} + (\ln m) V_T$$

$$= V_{be} + (-1.63\,mV)(\Delta T) + 18.4\,\frac{kT}{q}$$

$$= V_{be} + (-1.63m)(T - 300) + 18.4\,\frac{kT}{q}$$

$$\boxed{\begin{aligned} V_{out_{error}}(T = 273) &\approx 1.2\,mV \\ V_{out_{error}}(T = 358) &\approx 2.54\,mV \end{aligned}}$$

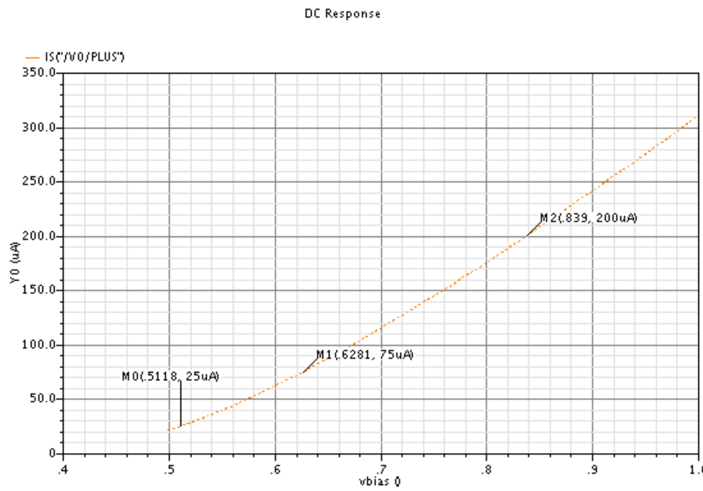
Problem. 7.5

b)

Bias: 25 uA/um => Vbias = 512mV,

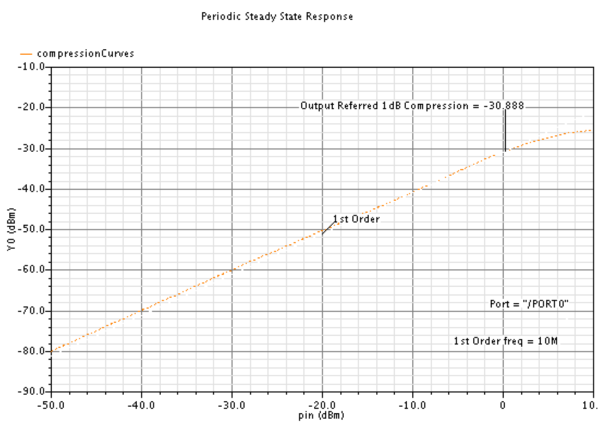
Bias: 75 uA/um => Vbias = 628mV

Bias: 200 uA/um => Vbias = 839mV

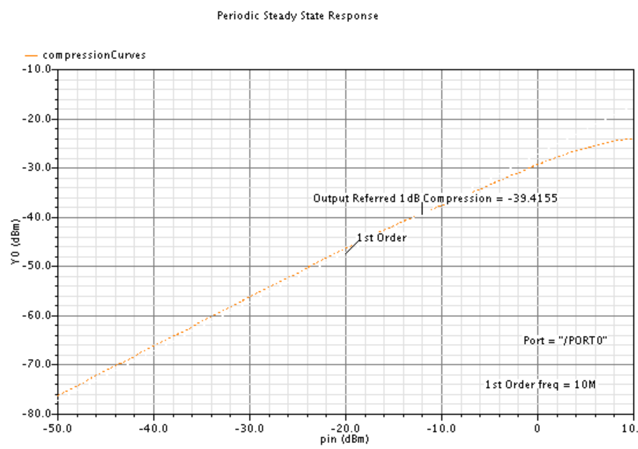


P1dB

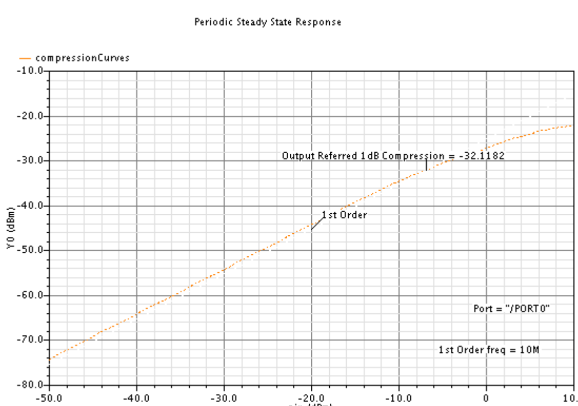
Bias: 25 uA/um



Bias: 75 uA/um

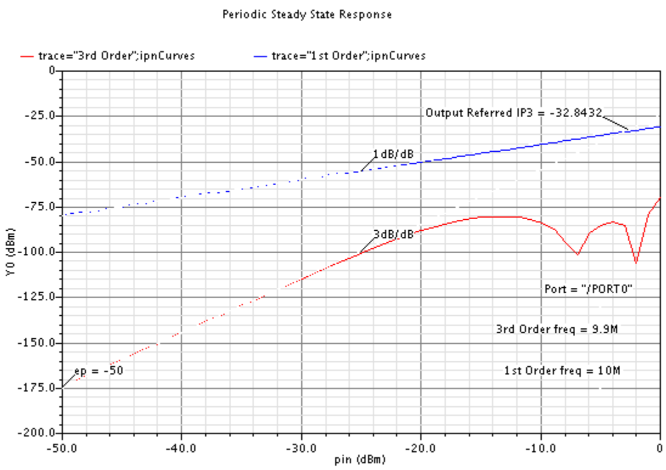


Bias: 200 uA/um

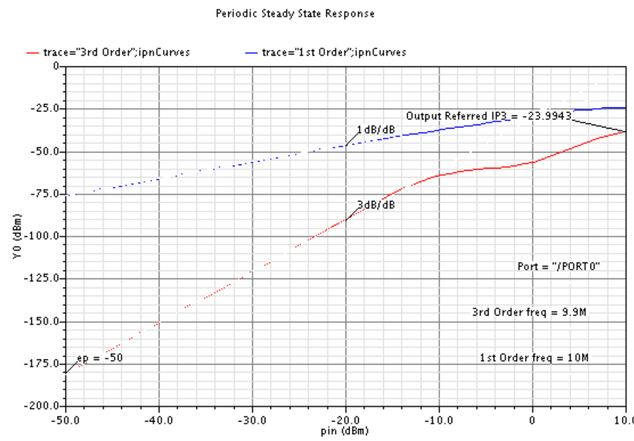


OIP3

Bias: 25 uA/um



Bias: 75 uA/um



Bias: 200 uA/um