Problem 7.1: For this problem, assume the LO is a square wave at 900MHz, and the RF signal is a tone at 1GHz.

a) Considering only the first harmonic component of the LO signal, what frequencies appear at the output of the mixer? Assume the mixer is perfectly balanced.

\[
\begin{align*}
(900 + 1000) \text{MHz} &= 1900 \text{ MHz} \\
(1000 - 900) \text{MHz} &= 100 \text{ MHz}
\end{align*}
\]

b) Derive an expression for the conversion gain of the mixer from the RF signal to the desired component at IF.

\[
G_c = \frac{\text{IF amplitude}}{\text{RF amplitude}} = \frac{\Delta f}{\pi f_0}
\]

c) Now introduce a phase offset between the LO applied to the left and right mixers of \(\Delta \theta\). Assume \(\Delta \theta\) is small. Derive an expression for the output amplitude of the LO leakage at 900MHz as a result of the offset.

Hint: \(\cos(A + \Delta) \approx \cos(A) - \Delta \sin(A)\) for small \(\Delta\).

Assume \(V_i = \frac{I_{RF}}{2\pi} = 0\) and consider only 1st harmonic

\[
i_{IF} = I_{Bias} \left( \frac{4}{\pi} \cos(w_{LO} t) \right) \quad i_{IF} = I_{Bias} \left( \frac{4}{\pi} \cos(w_{LO} + \Delta) t \right) \approx \frac{4I_{Bias}}{\pi} \left[ \cos(w_{LO} t) - \Delta \sin(w_{LO} t) \right]
\]

\[
I_{out} = i_{IF} + i_{IF}' = \frac{4}{\pi} I_{Bias} \Delta \sin(w_{LO} t)
\]

d) Using your expression for conversion gain from part b), find an expression for the ratio of LO leakage amplitude to IF signal amplitude in dBc. Assuming \(G_m v_{in} = 1\%\) of \(I_{Bias}\) for good linearity and \(\Delta \theta = 2\pi/100\) (1%), what is the required slope of the IF filter in dB/decade to attenuate the LO signal to -40dBc below the IF signal level?

\[
\frac{4}{\pi} \frac{I_{Bias} \Delta}{\frac{4}{\pi} G_m v_{in}} = 20 \Delta
\]

\[
\frac{20 \log(2)}{8.95} = -40 dBc
\]

\[
\frac{20 \log(2\pi) - (-40)}{\varepsilon - 8.95} = m
\]

\[
m = -58.65 \text{ dB/dec}
\]
Problem 7.2: You may neglect $r_o$ and base current ($\beta = \infty$) for this problem.

a) Simplify $L$, $C_1$, and $C_2$ into an equivalent parallel RLC tank. Use the series-to-parallel resistance transformations discussed in lecture to find an expression for the equivalent loss $R_{eq}$ of the parallel tank. Assume a series loss in the inductor $R_s$, and an inductor $Q_L = 10$. Do not neglect the impedance on node $V_x$ in your expression for $R_{eq}$.

\[ L' = L \left( \frac{Q_L^2 + 1}{Q_L^2} \right) \]

\[ R_L' = R_L (Q_L^2 + 1) \]

\[ Q_L = \frac{w R_L C_L}{\frac{Q_L^2}{Q_L^2 + 1}} \]

\[ C_2' = C_2 \left( \frac{Q_L^2}{Q_L^2 + 1} \right) \]

\[ R_L' = R_L \left( \frac{1}{Q_L^2 + 1} \right) \]

\[ Q_{12} = \frac{1}{w R_c' C_{12}} \quad C_{12} = \frac{C_1 C_2'}{C_1 + C_2'} \]

\[ C_{12}' = C_{12} \left( \frac{Q_L^2 + 1}{Q_{12}^2} \right) \]

\[ R_c' = R_c \left( \frac{1}{Q_L^2 + 1} \right) \]

\[ R_c'' = R_c' \left( \frac{Q_L^2}{Q_{12}^2} \right) \]

\[ R_{eq} = \frac{1}{2 \pi f_m} \left( \frac{C_1 + C_2}{C_1} \right)^2 \]

\[ Y_{eq} = \frac{1}{w L'} + \frac{1}{R_L'} + j w C_{12}' + \frac{1}{R_c''} = \left( \frac{1}{R_L'} + \frac{1}{R_c''} \right) + j \left( w C_{12}' - \frac{1}{w L'} \right) \]

\[ R_{eq} = \text{Re} \left( Y_{eq}^{-1} \right) \approx \left( \frac{1}{R_L'} + \frac{1}{R_c''} \right)^{-1} \quad \text{for} \quad w = w_{osc} \]

where \[ L' = \frac{101}{100} L \approx L \quad R_L' = 101 R_L \quad R_c'' = \frac{1}{2 \pi f_m} \left( \frac{C_1 + C_2'}{C_1} \right)^2 \]
b) Derive an expression for the oscillation frequency.

\[
f_{osc} = \frac{1}{2\pi} \frac{1}{\sqrt{L \cdot C_{12}}} \approx \frac{1}{2\pi \sqrt{\frac{C_1 + C_2}{L \cdot C_1 \cdot C_2}}} \quad \text{for} \quad Q_{12} \ll 1 \quad \text{and} \quad Q_c \gg 1
\]

For oscillation

\[
\frac{1}{n \beta_m} = K_E = \frac{\left( \frac{1}{R_c} + n^2 \beta_m \right)}{\left( \frac{1}{R_L} + n^2 \beta_m \right)} \quad \text{for} \quad w = w_{osc}
\]

\[
\beta_m = \frac{1}{(n-n^2)R_c} = \frac{1}{(n-n^2)(Q_c^2+1)R_L}
\]

\[
\beta_m = \frac{I_{Bias}}{V_T} = \frac{1}{n(1-n)(Q_L^2+1)R_L}
\]

\[
I_{Bias} = \frac{V_T}{n(1-n)(Q_c^2+1)R_L}
\]

where \( n = \frac{C_1}{C_1 + C_2} \)

\( Q_c = 10 \)

\( V_T = \frac{kT}{q} \)
Problem 7.3: For this problem, assume L and C are lossless, and all losses are modeled by \( R_{eq} \).

a) Derive an expression for the differential amplitude of the fundamental oscillation across the tank. Assume all higher-order harmonics are perfectly filtered by the tank.

\[
V_{osc} \approx I_{bias} \left( R_{eq} \right) \left( \frac{4}{\pi} \right) \cos \left( \omega_0 t \right)
\]

\[
|V_{osc}| \approx \left( \frac{4}{\pi} \right) \left( R_{eq} \cdot I_{bias} \right)
\]

b) Replace the NMOS devices with NPNs. Explain how the amplitude is limited by the parasitic diodes in the BJTs. How would you modify the circuit to eliminate this limitation?

→ Amplitude is limited due to current path through parasitic diodes.

→ To eliminate this effect you can add voltage dividers using capacitors.

→ The problem with this is that you will now need a biasing circuit for the BJTs.

→ You may also use buffers to reduce the limitation due to parasitic diodes.
Problem 7.4: This problem refers to the bandgap reference and corresponding expression for \( \frac{\partial V_{BE}}{\partial T} \) shown in Lecture 22.

a) Assuming \( V_{GO} = 1.2V, V_{BE} = 750mV \) at 300K, and from simulation, the exponent of \( I_s \) temperature dependency \( r = 2 \). What is the value of \( n \), the exponent of \( I_c \) temperature dependency, determined by the circuit on the right?

\[
V_{BE1} = V_{BE2} + I_c R
\]

\[
I_c = \frac{kT}{qR} \ln m \Rightarrow I_c \propto T^n
\]

Therefore, \( n = 1 \).

b) Using your answer from part a, calculate \( \frac{\partial V_{BE}}{\partial T} \) at 300K and determine the value of \( m \) (BJT area scaling ratio) to cancel this slope.

\[
\frac{dV_{BE}}{dT} = \frac{V_{GO} - V_{BE} + (r-n)V_T}{T} = \frac{1.2 - 0.75 + (2-1)(0.026)}{300} = -1.587 \text{ mV/K}
\]

\[
\frac{dAV_{BE}}{dT} = \frac{k}{q} \ln m = 1.587 m \Rightarrow \ln m = 18.4
\]

\[ m \approx 9.76 \times 10^7 \]

c) Now assume the resistor has a temperature dependency, typically proportional to \( \sqrt{T} \). What is the new value of \( n \), the exponent of \( I_c \) temperature dependency, and new value of \( \frac{\partial V_{BE}}{\partial T} \)?

\[
\text{From part a)} \quad I_c = \frac{kT}{qR} \ln m \Rightarrow I_c \propto \frac{T^{\frac{1}{2}}}{T^n}
\]

\[
I_c \propto T^{\frac{1}{2}} \Rightarrow n = \frac{1}{2}
\]

\[
\frac{dV_{BE}}{dT} = \frac{-V_{GO} - V_{BE} + (r-n)V_T}{T} = -1.63 \text{ mV/K}
\]
d) Using your value of \( m \) from part b), and the value of \( \partial V_{BE} / \partial T \) from part c) calculated at 300K, what is the error voltage at \( V_{out} \) at 0°C and 85°C, the extremes of the commercial temperature range?

\[
V_{0,\text{error}} = V_{be} + \Delta V_{be} + (\ln m) V_T
\]

\[
= V_{be} + (1.63 \text{ mV}) (\Delta T) + 18.4 \frac{kT}{q}
\]

\[
= V_{be} + (1.63 mV)(T - 300) + 18.4 \frac{kT}{q}
\]

\[
V_{out,\text{error}} \quad (T = 273) \approx 1.2 \text{ mV}
\]

\[
V_{out,\text{error}} \quad (T = 358) \approx 2.54 \text{ mV}
\]
Problem 7.5

b) Bias: 25 \mu A/\mu \text{m} \Rightarrow V_{\text{bias}} = 512 \text{mV},

Bias: 75 \mu A/\mu \text{m} \Rightarrow V_{\text{bias}} = 628 \text{mV}

Bias: 200 \mu A/\mu \text{m} \Rightarrow V_{\text{bias}} = 839 \text{mV}

P1dB

Bias: 25 \mu A/\mu \text{m}

Bias: 75 \mu A/\mu \text{m}

Bias: 200 \mu A/\mu \text{m}
OIP3

Bias: 25 uA/um

Bias: 75 uA/um

Bias: 200 uA/um