EECS 530 Fall 2006: Homework #3
Assigned: Thursday 21st September, 2006
Due: Thursday 28th September, 2006

Question 1: Cylindrical Coordinates

In cylindrical coordinates, we can define the following:

$$r = \sqrt{x^2 + y^2}$$
$$\varphi = \tan^{-1}(y/x)$$
$$z = z$$

a) Find the inverse relation for $x(r,\varphi,z)$, $y(r,\varphi,z)$, $z(r,\varphi,z)$.

b) Find the scaling factors h_r, h_φ, and h_z (also know as "metrical coefficients").

c) Define each of the following operations in Cylindrical Coordinates:

- $\nabla\psi$
- $\nabla\cdot\mathbf{F}$
- $\nabla\times\mathbf{F}$
- $\nabla^2\psi = \nabla\cdot\nabla\psi$

Confirm your results with those available in books.

Question 2: Elliptic Coordinates

Let two fixed points P_1 and P_2 be located at $x = c$ and $x = -c$ on the x-axis and let r_1 and r_2 be the distances of a variable point P in the xy-plane from P_1 and P_2. Let the axes of the coordinate system be defined as:

$$u_1 = \xi \quad u_2 = \eta \quad u_3 = z$$

Where

$$\xi = \frac{r_1 + r_2}{2c} \text{ and } \eta = \frac{r_1 - r_2}{2c}$$

Note that:

$$\xi \geq 1 \text{ and } -1 \leq \eta \leq 1$$

a) Find expressions for the Cartesian coordinates x and y in terms of ξ and η.

b) Let $z = 0$. Plot a few ξ = constant curves and η = constant curves to get a feel for the coordinate system. Note: Beware sign ambiguity; your plots should cover all four quadrants.

c) Find the scaling factors h_ξ, h_η, and h_z (also know as "metrical coefficients").

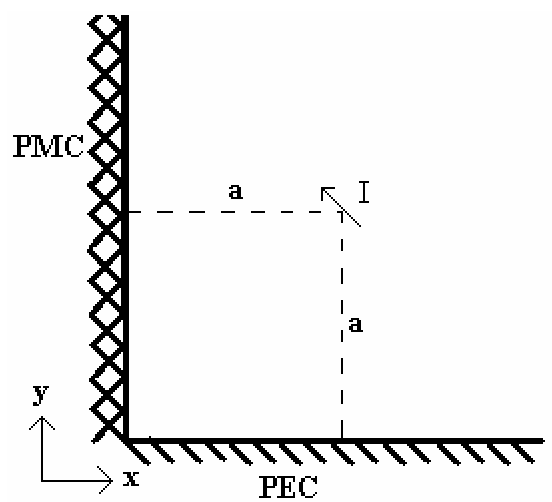
d) Define each of the following operations in Elliptic Coordinates:

- $\nabla \psi$
- $\nabla \cdot \mathbf{F}$
- $\nabla \times \mathbf{F}$
- $\nabla^2 \psi = \nabla \cdot \nabla \psi$

Question 3: Image Theory at a Corner

Consider a 2D space, infinite in the z-direction, which is defined by the perpendicular intersection of semi-infinite PEC and PMC planes. Let a point source, $\mathbf{J}$, exist in this space. If the origin is defined by the intersection of the two planes then the expression for the current is given by:

$$\mathbf{J} = J_0 \frac{\hat{\mathbf{y}} - \hat{\mathbf{x}}}{\sqrt{2}} \delta(x-a, y-a, z)$$



a) Draw the equivalent image currents.

b) Suppose $\mathbf{J}$ is replaced with $\mathbf{J_m}$, a magnetic current density. Draw the equivalent image currents.

Question 4: Levitation

Consider a loop, located at the origin and placed on the xy-plane. It has a radius a, with N turns and carries a current of I, moving counter-clockwise as seen from +z axis. The magnetic field due to the loop can be expressed as:

$$\overline{\mathbf{H}} = \frac{NIa^2}{4R^3}(\hat{\mathbf{R}} 2\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta)$$

Use the modified Lorentz law, and image theory to calculate the force on the loop if it is held above a perfect electric conductor at height z_o. Hint: Place the image loop at the origin. Also, note: assume that the current is time-variant enough to be levitated, but static enough to treat it as such.

$$\overline{\mathbf{F}} = NI\oint_C d\overline{\mathbf{l}} \times \overline{\mathbf{B}}$$

Question 5: Duality & Boundary Conditions

A magnetic sphere with relative permeability μ_r and relative permittivity $\varepsilon_r = 1$ rests in free space. A static magnetic field of $H_0\hat{\mathbf{z}}$ lies within this sphere. Assume no currents or charges exist on the surface of the sphere. Find the magnetic field on the outside surface of the sphere using.

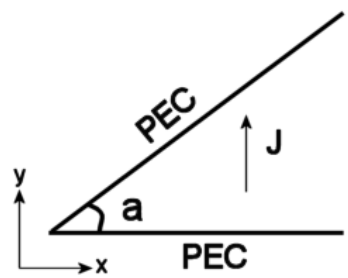
a) Duality with your answer from HW 2, Question 5
b) Boundary Conditions

Verify that you get the same answer using both methods.

Question 6: Image Theory at a Wedge

Consider a 2D space, infinite in the z-direction, as in question 2. Two PEC planes (both perpendicular to the xy-plane) intersect at an angle a. Current **J** exists in the space, given by the following expression.

$$\mathbf{J} = J_0 \hat{\mathbf{y}} \delta(x-b, y-b, z)$$



a) Suppose a = 45°. Draw the equivalent image currents.
b) Suppose a = 44.5°. Draw the equivalent image currents.
c) Comparing your answer to part b) and c), what can you generalize about applying image theory to wedges with angle a = 360/n or a ≠ 360/n? (n integer)