Homework submission policy: same as in previous weeks

1. Exercise 6.23, p. 6-30

2. (a) Find the rates and minimum distances of the orthogonal, biorthogonal and hypercube codes for the AGC, assuming each has $2^M$ codewords with Euclidean lengths $L$.
   (b) Make a plot of their minimum distances vs. their rates, assuming $M = 3$ and $L = 1$.
   (c) Use the results of part (a) to order the codes according to their rates.
   (d) Use the results of part (a) to order the codes according to their error probabilities, assuming optimum decoders are used and $L/\sigma_V$ is large.
   (e) Find upper and lower bounds to their error probabilities.

3. Exercise 6.38, p. 6-32. Replace (b) with “Compare the performance of this code to the performances of the orthogonal, biorthogonal and hypercube codes.”

4. Exercise 6.29, p. 6-33

5. Exercise 6.30, p. 6-33. In part (b), replace “$c/\sigma^2_V$” with “$a^2/\sigma^2_V$.”

6. Exercise 7.5, p. 7-15

7. Exercise 7.6, p. 7-15

8. Exercise 7.7, p. 7-15

9. A binary asymmetric channel has alphabets $A_X = A_Y = \{0,1\}$ and transition probability distribution $q(1|0) = 0.1$ and $q(0|1) = 0.2$. Let $N$ be a large integer.
   (a) Find, approximately, the size of the fanout from a channel input sequence of length 100 containing fifty ones.
   (b) Of channel input sequences with length $N$, which have the smallest fanout? Which have the largest fanout?
   (c) Given that a channel input distribution $p_i$ with $p_i(0) = \alpha$, find the corresponding output probability distribution $p_Y$, and find, approximately, the number of output sequences that are typical of $p_Y$.
   (d) What value of $\alpha$ maximizes the number of typical output sequences?
   (e) Given that $p_i(0) = \alpha$, find an approximate expression for the ratio of the size of the set of channel output sequences typical wrt $p_Y$ to the size of a fanout from a sequence that is typical of $p_i$.
   (f) What value of $\alpha$ maximizes the ratio found in part (e)?
   (g) What is the capacity of this channel?