EECS 555 Problem Set 2

Due: Friday, January 28. Reading: Chapter 3,4 and Appendix C.

1. (a) Show that for any equienergy, equidistant (real) signal set, (s_i, s_j) = a constant for $i \neq j$. (Note: equienergy implies $\| s_i \|^2$ is a constant and equidistant implies $\| s_i - s_j \|$ is a constant).

   (b) For any equienergy signal set show that

$$\rho_{ave} = \frac{1}{M(M-1)} \sum_{i=0}^{M-1} \sum_{\substack{j=0 \\ j \neq i}}^{M-1} \rho_{ij} \geq -\frac{1}{M-1}$$

   where

$$\rho_{ij} = (s_i, s_j)/(||s_i||||s_j||) = (s_i, s_j)/E$$

2. A set of 16 signals is constructed in 7 dimension using only two possible coefficients, i.e. $s_{i,j} \in \left\{+\sqrt{E}, -\sqrt{E}\right\}$. Let $A_k = |\{(i,j) : |s_i - s_j|^2 = 4Ek\}|$ i.e. A_k is the number of signal pairs with squared distance $4Ek$. The signals are chosen so that

$$A_k = \begin{cases} 16 & k = 0 \\ 0 & k = 1, 2 \\ 112 & k = 3, 4 \\ 0 & k = 5, 6 \\ 16 & k = 7 \end{cases}$$

   Find the union bound and the union-Bhattacharyya bound on the error probability of the optimum receiver in additive white Gaussian noise with two sided power spectral density $N_0/2$.

3. Consider a digital communication system that transmits one of two equally likely signals over a discrete time additive channel with nonwhite Gaussian noise. The two signals transmitted are

$$\boldsymbol{s_0} = (1, 1, -1)$$

   and

$$\boldsymbol{s_1} = (1, -1, 1).$$

   The noise added to each component of the signal is Gaussian but not independent nor identically distributed. The noise vector $\boldsymbol{n} = (n_0, n_1, n_2)$ has zero mean and covariance matrix.

$$K = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

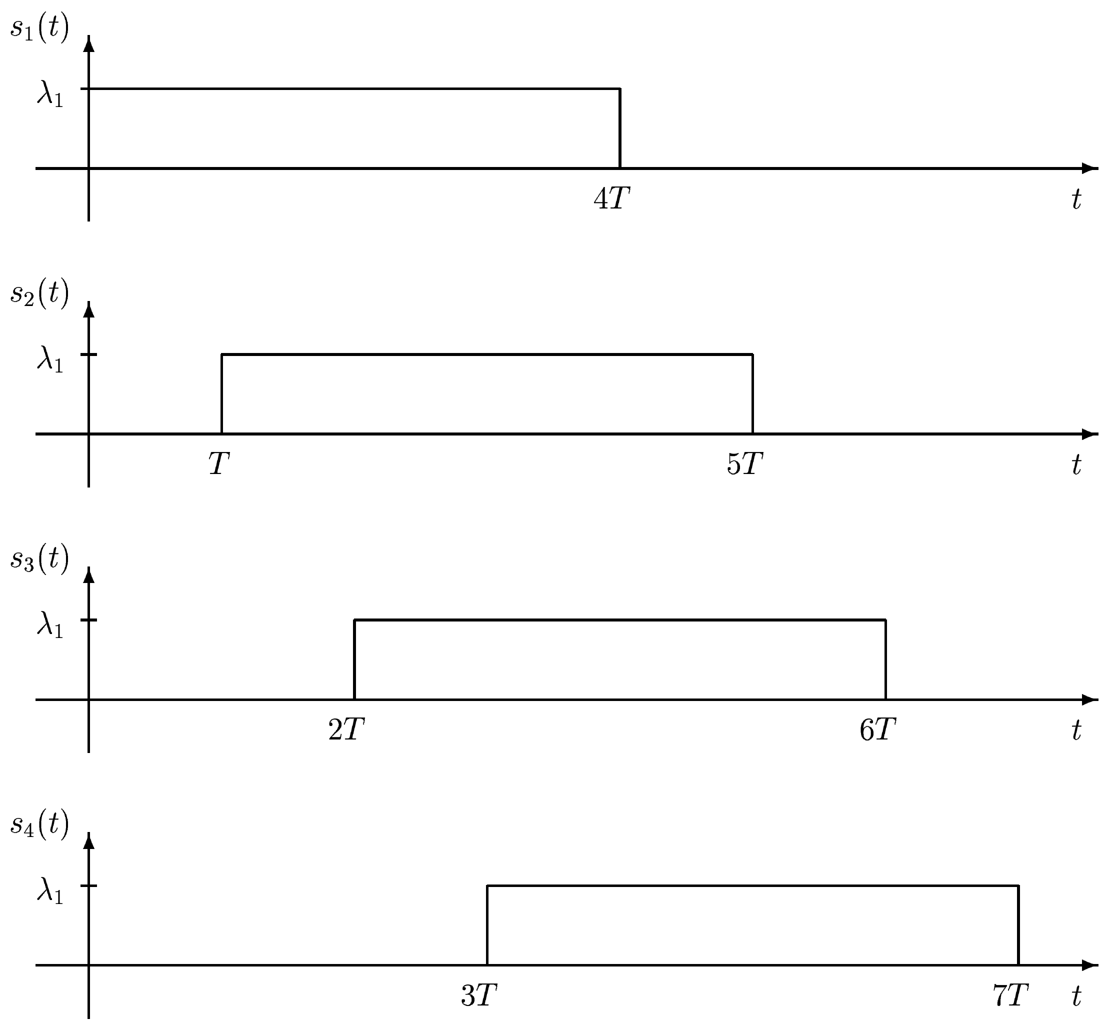
   The covariance matrix has eigenvalues, $\lambda_1 = 5$, $\lambda_2 = 5$ and $\lambda_3 = 1$ with corresponding (orthonormal) eigenvectors

$$\begin{aligned} \phi_1 &= (-1/\sqrt{2}, 1/\sqrt{2}, 0), \\ \phi_2 &= (0, 0, 1), \\ \phi_3 &= (1/\sqrt{2}, 1/\sqrt{2}, 0). \end{aligned}$$

(a) Find the optimal receiver for minimizing the error probability between the two signals.

(b) Find the error probability for the optimal receiver.

4. Consider a Poisson channel with one of 4 signals transmitted. Let the signals be as shown below. Assume when the signal is present that the intensity of the photon process is λ_1 and when the signal is not present the intensity is λ_0. That is the received signal during the interval $[0, T]$ is Poisson with parameter λ_1 if the laser is on and λ_0 if the laser is off. Find the optimal receiver for minimizing the probability of error for a signal (as opposed to a bit). Find an upper bound on the error probability.



5. A signal set consists of 256 signals in 16 dimensions with the coefficients being either $+\sqrt{E}$ or $-\sqrt{E}$. The distance structure is given as

$$A_k = |\{(i,j) : ||s_i - s_j||^2 = 4Ek\}| = \begin{cases} 256 & k = 0 \\ 28672 & k = 6 \\ 7680 & k = 8 \\ 28672 & k = 10 \\ 256 & k = 16 \\ 0 & \text{otherwise} \end{cases}$$

These signals are transmitted with equal probability over an additive white Gaussian noise channel. Determine the union bound on the error probability. Determine the union-Bhattacharyya bound on the error probability. Express your answer in terms of the energy transmitted per bit. What is the rate of the code in bits/dimension?