# EECS 583 - Class 8 Static Single Assignment Form Classic Optimization 

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## Static Single Assignment (SSA) Form

* Difficulty with optimization
» Multiple definitions of the same register
» Which definition reaches
» Is expression available?

* Static single assignment
» Each assignment to a variable is given a unique name
» All of the uses reached by that assignment are renamed
» DU chains become obvious based on the register name!


## Converting to SSA Form

* Trivial for straight line code

$$
\begin{aligned}
& x=-1 \\
& y=x \\
& x=5 \\
& z=x
\end{aligned}
$$



* More complex with control flow - Must use Phi nodes

$$
\begin{aligned}
& \text { if }(\ldots) \\
& x=-1 \\
& \text { else } \\
& \quad x=5 \\
& y=x
\end{aligned}
$$

```
if (... )
    \(\mathrm{x} 0=-1\)
else
    \(\mathrm{x} 1=5\)
\(\mathrm{x} 2=\operatorname{Phi}(\mathrm{x} 0, \mathrm{x} 1)\)
\(y=x 2\)
```


## Converting to SSA Form (2)

* What about loops?
» No problem!, use Phi nodes again



## SSA Plusses and Minuses

* Advantages of SSA
» Explicit DU chains - Trivial to figure out what defs reach a use
$\ddot{Y}$ Each use has exactly 1 definition!!!
» Explicit merging of values
» Makes optimizations easier
* Disadvantages
» When transform the code, must either recompute (slow) or incrementally update (tedious)


## Phi Nodes (aka Phi Functions)

* Special kind of copy that selects one of its inputs
* Choice of input is governed by the CFG edge along which control flow reached the Phi node

$\%$ Phi nodes are required when 2 non-null paths $\mathrm{X} \rightarrow \mathrm{Z}$ and $\mathrm{Y} \rightarrow \mathrm{Z}$ converge at node Z , and nodes X and Y contain assignments to V


## SSA Construction

* High-level algorithm

1. Insert Phi nodes
2. Rename variables

* A dumb algorithm
» Insert Phi functions at every join for every variable
» Solve reaching definitions
" Rename each use to the def that reaches it (will be unique)
* Problems with the dumb algorithm
» Too many Phi functions (precision)
» Too many Phi functions (space)
" Too many Phi functions (time)


## Need Better Phi Node Insertion Algorithm

 for some predecessors p of m

def in BB4 forces Phi in BB6 def in BB6 forces Phi in BB7 def in BB7 forces Phi in BB1

Phi is placed in the block that is just outside the dominated region of the definition BB

Dominance frontier
The dominance frontier of node X is the set of nodes Y such that

* X dominates a predecessor of Y , but
* X does not strictly dominate Y


## Recall: Dominator Tree

First BB is the root node, each node dominates all of its descendants


| BB | DOM | BB | DOM |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 4 | $0,1,3,4$ |
| 1 | 0,1 | 5 | $0,1,3,5$ |
| 2 | $0,1,2$ | 6 | $0,1,3,6$ |
| 3 | $0,1,3$ | 7 | $0,1,7$ |



Dom tree

## Computing Dominance Frontiers



DF

For each join point X in the CFG
For each predecessor, Y , of X in the CFG Run up to the $\operatorname{IDOM}(\mathrm{X})$ in the dominator tree, adding X to $\mathrm{DF}(\mathrm{N})$ for each N between Y and $\operatorname{IDOM}(\mathrm{X})$ (or X , whichever is encountered first)

## Class Problem

Compute dominance frontiers for each BB


Dominator Tree


For each join point X in the CFG
For each predecessor, Y, of X in the CFG
Run up to the $\operatorname{IDOM}(\mathrm{X})$ in the dominator tree, adding X to $\mathrm{DF}(\mathrm{N})$ for each N between Y and $\operatorname{IDOM}(\mathrm{X})$ (or X , whichever is encountered first)

## SSA Step 1 - Phi Node Insertion

* Compute dominance frontiers
\% Find global names (aka virtual registers)
» Global if name live on entry to some block
» For each name, build a list of blocks that define it
* Insert Phi nodes
» For each global name $n$
$\ddot{Y}$ For each BB b in which n is defined
- For each BB d in b's dominance frontier
o Insert a Phi node for n in d
o Add d to n's list of defining BBs


## Phi Node Insertion - Example



## Class Problem

Insert the Phi nodes


Dominance frontier


| BB | DF |
| :--- | :--- |
| 0 | - |
| 1 | - |
| 2 | 4 |
| 3 | 4,5 |
| 4 | 5 |
| 5 | 1 |

## SSA Step 2 - Renaming Variables

* Use an array of stacks, one stack per global variable (VR)
* Algorithm sketch
» For each BB b in a preorder traversal of the dominator tree
$\ddot{Y}$ Generate unique names for each Phi node
$\ddot{Y}$ Rewrite each operation in the BB
- Uses of global name: current name from stack
- Defs of global name: create and push new name
$\ddot{Y}$ Fill in Phi node parameters of successor blocks
$\ddot{Y}$ Recurse on b's children in the dominator tree
$\ddot{Y}<$ on exit from $b>$ pop names generated in $b$ from stacks


## Renaming - Example (Initial State)



```
var: a b c d i
ctr: 0
stk: a0 b0 c0 d0 i0
```


## Renaming - Example (After BB0)



```
var: a b c d i
ctr: 1 1 1 1 1
stk: a0 b0 c0 d0 i0
```


## Renaming - Example (After BB1)



```
var: a b c d i
ctr: 3 2 3 2 2
stk: a0 b0 c0 d0 i0
    al bl cl d1 il
    a2 c2
```


## Renaming - Example (After BB2)



```
var: a b c d i
ctr: 3 3 4 4 3 2
stk: a0 b0 c0 d0 i0
    al b1 cl d1 il
    a2 b2 c2 d2
    c3
```


## Renaming - Example (Before BB3)



```
var: a b c d i
ctr: 3 3 4 4 3 2
stk: a0 b0 c0 d0 i0
    a1 b1 c1 d1 il
    a2 c2
```


## Renaming - Example (After BB3)



```
var: a b c d i
ctr:4 3 4 4 2
stk: a0 b0 c0 d0 i0
    al b1 cl d1 il
    a2 c2 d3
    a3
```


## Renaming - Example (After BB4)



```
var: a b c d i
ctr:4 3 4 5 2
stk: a0 b0 c0 d0 i0
    al bl cl d1 il
a2 c2 d3
a3
d4
```


## Renaming - Example (After BB5)



```
var: a b c d i
ctr:4 3 5 5 2
stk: a0 b0 c0 d0 i0
    al b1 cl d1 il
a2 c2 d3
a3 c4
```


## Renaming - Example (After BB6)



```
var:a b c d i
ctr: 4 4 6 6
stk: a0 b0 c0 d0 i0
    a1 b1 c1 d1 il
    a2 b3 c2 d3
    a3 c5 d5
```


## Renaming - Example (After BB7)



```
var: a b c d i
ctr: 5 5 7 7 3
stk: a0 b0 c0 d0 i0
    al b1 c1 d1 il
    a2 b4 c2 d6 i2
    a4
c6
```

Fin!

## Class Problem

Rename the variables
Dominance frontier


| BB | DF |
| :--- | :--- |
| 0 | - |
| 1 | - |
| 2 | 4 |
| 3 | 4,5 |
| 4 | 5 |
| 5 | 1 |

## Code Optimization

## Code Optimization

\% Make the code run faster on the target processor
» Other objectives: Power, code size

* Classes of optimization
» 1. Classical (machine independent)
$\ddot{Y}$ Reducing operation count (redundancy elimination)
Y Simplifying operations
Y Generally good for any kind of machine
» 2. Machine specific
$\ddot{Y}$ Peephole optimizations
$\ddot{Y}$ Take advantage of specialized hardware features
» 3. Parallelism enhancing
$\ddot{Y}$ Increasing parallelism (ILP or TLP)
$\ddot{Y}$ Possibly increase instructions


## A Tour Through the Classical Optimizations

* For this class - Go over concepts of a small subset of the optimizations
» What it is, why its useful
» When can it be applied (set of conditions that must be satisfied)
» How it works
» Give you the flavor but don't want to beat you over the head
* Challenges
» Register pressure?
» Parallelism verses operation count


## Dead Code Elimination

* Remove any operation who's result is never consumed
* Rules
» X can be deleted
$\ddot{Y}$ no stores or branches
» DU chain empty or dest register not live
* This misses some dead code!!
» Especially in loops
» Critical operation
$\ddot{Y}$ store or branch operation
» Any operation that does not directly or indirectly feed a critical operation is dead
» Trace UD chains backwards from critical operations
» Any op not visited is dead



## Local Constant Propagation

* Forward propagation of moves of the form
» $\mathrm{rx}=\mathrm{L}$ (where L is a literal)
» Maximally propagate
* Consider 2 ops, X and Y in a $\mathrm{BB}, \mathrm{X}$ is before Y
» 1. X is a move
" 2. $\operatorname{src} 1(\mathrm{X})$ is a literal
» 3. Y consumes $\operatorname{dest}(\mathrm{X})$
» 4. There is no definition of $\operatorname{dest}(\mathrm{X})$ between X and Y
» 5. No danger betw X and Y
$\ddot{Y}$ When $\operatorname{dest}(X)$ is a Macro reg, BRL destroys the value

$$
\mathrm{rl}=5
$$

$$
\mathrm{r} 2=\text { ' } \quad \mathrm{x} \text { ' }
$$

$$
\mathrm{r} 3=7
$$

$$
\mathrm{r} 4=\mathrm{r} 4+\mathrm{r} 1
$$

$$
\mathrm{r} 1=\mathrm{r} 1+\mathrm{r} 2
$$

$$
\mathrm{rl}=\mathrm{r} 1+1
$$

$$
\mathrm{r} 3=12
$$

$$
\mathrm{r} 8=\mathrm{r} 1-\mathrm{r} 2
$$

$$
\mathrm{r} 9=\mathrm{r} 3+\mathrm{r} 5
$$

$$
\mathrm{r} 3=\mathrm{r} 2+1
$$

$$
\mathrm{r} 10=\mathrm{r} 3-\mathrm{r} 1
$$

Note, ignore operation format issues, so all operations can have literals in either operand position

## Global Constant Propagation

* Consider 2 ops, X and Y in different BBs
» 1. X is a move
» 2. $\operatorname{src} 1(\mathrm{X})$ is a literal
» 3 . Y consumes $\operatorname{dest}(\mathrm{X})$
» 4. X is in a_in( $\mathrm{BB}(\mathrm{Y}))$
» 5. $\operatorname{Dest}(\mathrm{x})$ is not modified between the top of $\mathrm{BB}(\mathrm{Y})$ and Y
» 6. No danger betw X and Y
$\ddot{Y}$ When $\operatorname{dest}(\mathrm{X})$ is a Macro reg, BRL destroys the value



## Constant Folding

* Simplify 1 operation based on values of src operands
» Constant propagation creates opportunities for this
* All constant operands
» Evaluate the op, replace with a move
$\ddot{Y} \mathrm{rl}=3 * 4 \rightarrow \mathrm{rl}=12$
$\ddot{\mathrm{Y}} \mathrm{rl}=3 / 0 \rightarrow$ ??? Don't evaluate excepting ops !, what about floating-point?
» Evaluate conditional branch, replace with BRU or noop
$\ddot{Y}$ if $(1<2)$ goto $\mathrm{BB} 2 \rightarrow$ BRU BB2
$\ddot{Y}$ if $(1>2)$ goto $\mathrm{BB} 2 \rightarrow$ convert to a noop
* Algebraic identities

```
» \(\mathrm{r} 1=\mathrm{r} 2+0, \mathrm{r} 2-0, \mathrm{r} 2 \mid 0, \mathrm{r}^{\wedge} 0, \mathrm{r} 2 \ll 0, \mathrm{r} 2 \gg 0\)
    \(\ddot{\mathrm{Y}} \mathrm{rl}=\mathrm{r} 2\)
» \(\mathrm{r} 1=0\) * \(\mathrm{r} 2,0 / \mathrm{r} 2,0\) \& r 2
    \(\ddot{\mathrm{Y}} \mathrm{rl}=0\)
» \(\mathrm{r} 1=\mathrm{r} 2 * 1, \mathrm{r} 2 / 1\)
    \(\ddot{Y} \mathrm{rl}=\mathrm{r} 2\)
```


## Class Problem



## Forward Copy Propagation

* Forward propagation of the RHS
of moves

$$
\begin{array}{ll}
» & \mathrm{r} 1=\mathrm{r} 2 \\
» & \cdots \\
» & \mathrm{r} 4=\mathrm{r} 1+1 \rightarrow \mathrm{r} 4=\mathrm{r} 2+1
\end{array}
$$

* Benefits
» Reduce chain of dependences
» Eliminate the move
* Rules (ops X and Y)
» X is a move
» $\operatorname{src} 1(\mathrm{X})$ is a register
» Y consumes $\operatorname{dest}(\mathrm{X})$

» X .dest is an available def at Y
» X.src1 is an available expr at Y


## CSE - Common Subexpression Elimination

* Eliminate recomputation of an expression by reusing the previous result

$$
\begin{array}{ll}
» & \mathrm{r} 1=\mathrm{r} 2 * \mathrm{r} 3 \\
> & \rightarrow \mathrm{r} 100=\mathrm{r} 1 \\
» & \ldots \\
» & \mathrm{r} 4=\mathrm{r} 2 * \mathrm{r} 3 \rightarrow \mathrm{r} 4=\mathrm{r} 100
\end{array}
$$

\% Benefits
» Reduce work
» Moves can get copy propagated
$\therefore \quad$ Rules (ops X and Y)
» X and Y have the same opcode
» $\operatorname{src}(\mathrm{X})=\operatorname{src}(\mathrm{Y})$, for all srcs
» $\operatorname{expr}(\mathrm{X})$ is available at Y
" if X is a load, then there is no store that may write to address(X) along any path between X and Y

if op is a load, call it redundant load elimination rather than CSE

## Class Problem



Optimize this applying

1. dead code elimination
2. forward copy propagation
3. CSE

## Loop Invariant Code Motion (LICM)

* Move operations whose source operands do not change within the loop to the loop preheader
» Execute them only 1 x per invocation of the loop
» Be careful with memory operations!
" Be careful with ops not executed every iteration



## LICM (2)

* Rules
» X can be moved
» $\operatorname{src}(\mathrm{X})$ not modified in loop body
» X is the only op to modify $\operatorname{dest}(\mathrm{X})$
» for all uses of $\operatorname{dest}(\mathrm{X}), \mathrm{X}$ is in the available defs set
» for all exit BB , if $\operatorname{dest}(\mathrm{X})$ is live on the exit edge, X is in the available defs set on the edge
» if X not executed on every iteration, then X must provably not cause exceptions
» if X is a load or store, then there are no


Homework 2 eliminates the last rule. You can also ignore the executed on every iteration rule for SpecLICM.


## Global Variable Migration

* Assign a global variable temporarily to a register for the duration of the loop
» Load in preheader
» Store at exit points
* Rules
» X is a load or store
» address( X ) not modified in the loop
» if X not executed on every iteration, then X must provably not cause an exception
" All memory ops in loop whose address can equal address(X) must always have the same address as X


